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DEVELOPING A FRAMEWORK OF HYBRID METHOD FOR TACKLING LARGE-SCALE MIXED INTEGER NONLINEAR PROGRAMMING PROBLEMS

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ABSTRACT

Mixed Integer Nonlinear Programming (MINLP) is one of the most general modeling paradigms in optimization which includes both nonlinear programming (NLP) and Mixed Integer Linear Programming (MILP). There are two types of hybrids methods for solving MINLP, such as hybrids between heuristic methods and other heuristic methods and hybrid methods between exact methods and heuristic methods. This study discusses the second hybrid method. In the proposed algorithmic stages, we will determine the search method and the variables to find an optimal solution. The hybrid method aims to gain computational performance or conceptual simplification, potentially at the cost of accuracy or precision.

Keywords: MINLP, large-scale, optimization, framework, hybrid

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1. INTRODUCTION

An optimization problem consists of the set of independent variables and parameters, and often includes conditions or restriction of the value of the variables [1]–[3]. Such restrictions are termed constraints of the problem. The other important component of an optimization problem is the "good" measure called the objective function that depends on the variables of the problem. The solution of the optimization problem is a set of variables that satisfy all constraints, in such a way that the objective function achieves an optimal value. The representative form of standard in expressing optimization issues, making it easier to solve the problem. The commonly used form is the objective function f and the constraint function g which is a real-value scalar function.

There are many optimization issues that contain discrete or integer variables [4]. These variables can occur in modeling as whole unit statements, such as variables on the determination of the number of labour, or decision models in which there are binary variables, such as variables for determining investment on portfolio issues. Continuous variables can also appear which for example present labour time, production volume. Nonlinearity may arise in the optimization model, for example for problems in which physical properties are present, such as the balance of fluid concentrations, or perhaps in the case of economies of scale. The optimization model for nonlinear problems with discrete and continuous variables is known as the Mixed Integer Nonlinear Programming (MINLP) model [5].

The special class of MINLP issues discussed in this study is to assume discrete variables that are linear and separable from continuous variables. This problem can be written as follows.

Minimize:

$$z = c^T y + f(x) \quad (1)$$

Subject to:

$$h(x) \leq 0 \quad (2)$$

$$g(x) + by \leq 0 \quad (3)$$

$$x \in X \subset \mathbb{R}_+^n, y \in Y \subset \mathbb{R}_+^n \quad (4)$$

Where$f: \mathbb{R}^n \to \mathbb{R}$ and$h: \mathbb{R}^n \to \mathbb{R}^p$, $g: \mathbb{R}^n \to \mathbb{R}^q$ are continues function and generally behave smoothly defined in the n-dimensional convex polyhedral expressed by $X = \{x: x \in X \subset \mathbb{R}_+^n, A_1 x \leq a_1\}; U = \{y: y \in Y \subset \mathbb{R}_+^n, A_2 y \leq a_2\}$.

The heuristic approach proposed in the literature to solve the MINLP problem includes the Variable Neighborhood Search [6][7], automatic variable assignment strategy [8], Local Branching, feasible neighborhood search [9], feasibility pump [10] and [11], heuristic based on iterative Rounding [12] proposes a MINLP heuristic called the Relaxed-Exact-Continuous-Integer Problem Exploration (RECIPE) algorithm. This algorithm combines global search phases based on the Neighborhood Search Variables [13] and local search phases. However, in the heuristic approach, one of the major algorithmic difficulties regarding to solving MINLP is finding a viable solution. From the standpoint of the worst case complexity, finding a decent MINLP solution is just as difficult as finding a viable Nonlinear Programming solution, the NP-hard [9].

2. MIXED INTEGER NONLINIER PROGRAMMING

MINLP refers to mathematical programming with continuous and discrete variables and nonlinearities in the objective function and constraints. Mixed-integer nonlinear programming (MINLP) problems combine the combinatorial difficulty of optimizing over discrete variable sets with the challenges of handling nonlinear functions. MINLP is one of the most general modeling paradigms in optimization and includes both nonlinear programming (NLP) and mixed-integer linear programming (MILP) as sub-problems. MINLPs are expressed as:

$$\begin{cases} \text{minimize}_x \quad f(x) \\ \textit{subject to} \quad c(x) \le 0 \\ \qquad x \in X \\ \qquad x_i \in \mathbb{Z}, \forall_i \in I \end{cases} \tag{5}$$

Where $f : \mathbb{R}^n \rightarrow \mathbb{R}$ and $c : \mathbb{R}^n \rightarrow \mathbb{R}^m$ are assumed to be twice continuously differentiable functions, $X \subset \mathbb{R}^n$ is a bounded polyhedral set, and $I \subseteq \{1, \dots, n\}$ is the index set of integer variables. Note that the model might include maximizing, or equality constraints, or lower and upper bounds $l \le c(x) \le u$.

Definition 1. Problem (5) is a convex MINLP if the problem functions $f(x)$ and $c(x)$ are convex. If either $f(x)$ or any $c_i(x)$ is a non convex, then that (5) is a non-convex MINLP.

Definition 2. Given a set *S*, the convex hull of *S* is denoted by *conv(S)* and defined as

$$conv(S) := \{x : x = \lambda x^{(1)} + (1 - \lambda)x^{(0)}, \forall 0 \le \lambda \le 1, \forall x^{(0)}, x^{(1)} \in S\}$$

If $X = \{x \in \mathbb{Z}^p : l\, x \le u\}$ and $l \in \mathbb{Z}^p, u \in \mathbb{Z}^p$, then $conv(X) = [l, u]$ is simply the hypercube. In general, however, even tough *X* itself is polyhedral, it is not easy to find *conv(X)*. The convex hull plays an important role in mixed-integer linear programming because a linear programming (LP) can solve a MILP by solving an LP over its convex hull. Unfortunately, finding the convex hull of a MILP is just as hard as solving the MILP.

The same result does not hold for MINLP, as the following example illustrates:

$$\underset{x}{\text{minimize}} \sum_{i=1}^{n} \left(x_i - \frac{1}{2}\right)^2$$

$$\text{subject to } x_i \in \{0,1\}$$

The solution of the continuous relaxation is $x = (\frac{1}{2} \dots \frac{1}{2})$, which is not an extreme point of the feasible set and in fact lies in the strict interior of the MINLP; see Figure 1. Because the continuous minimizer lies in the interior of the convex hull of the feasible integer set, it cannot be separated from the feasible set. However, (5) can be reformulated by introducing an objective variable η and a constraint $\eta \ge f(x)$. then the following equivalent MINLP is obtained.

$$\begin{cases} \min\limits_{\eta, x} \quad \eta \\ \text{subject to} \quad f(x) \le \eta \\ \qquad c(x) \le 0 \\ \qquad x \in X \\ \qquad x_i \in \mathbb{Z}, \forall_i \in I \end{cases} \tag{6}$$

The interesting point of (6) is that the optimal solution always lies on the boundary of the convex hull of the feasible set and therefore allows us to use cutting-plane techniques.

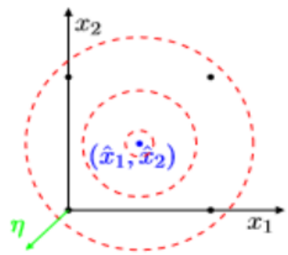

Figure 1. Small MINLP to illustrate the need for a linear objective function.

2.1. Relaxations of MINLPs

Here are strategies to be used to obtain relaxations of MINLPs.

1. Relaxing integrality. Integrality constraints $x_i \in \mathbb{Z}$ can be relaxed to $x_i \in \mathbb{R}$ for all $i \in I$. This procedure yields a nonlinear relaxation of MINLP. The type of relaxation can be found in branch and-bound algorithms and is given by:

$$\begin{cases} \min\limits_{x} & f(x) \\ \text{subject to} & c(x) \le 0 \\ & x \in X \end{cases} \tag{7}$$

2. Relaxing convex constraints. Constraints $c(x) \le 0$ and $f(x) \le z$ containing convex functions c and f can be relaxed with a set of supporting hyperplanes obtained from first-order Taylor series approximation:

$$z \ge f^{(k)} + \nabla f^{(k)^T}\left(x - x^{(k)}\right), \tag{8}$$

$$0 \ge c^{(k)} + \nabla c^{(k)^T}\left(x - x^{(k)}\right) \tag{9}$$

for a set of points $x^{(k)}, k = 1, \dots, K$. When c and f are convex, any collection of such hyperplanes forms a polyhedral relaxation of these constraints. This class of relaxations is used in the outer approximation methods.

3. Relaxing nonconvex constraints. Constraints $c(x) \le 0$ and $f(x) \le z$ containing nonconvex functions require more work to be relaxed. One approach is to derive convex underestimators, $\check{f}(x)$ and $\check{c}(x)$, which are convex functions that satisfy:

$$\check{f}(x) \le f(x) \text{ And } \check{c}(x) \le c(x), \forall x \in conv(X) \tag{10}$$

All these relaxations can enlarge the feasible set of (6), however they can be combined one with another. For example, a convex underestimator of a non-convex function can be further relaxed by using supporting hyperplanes, yielding a polyhedral relaxation. Figure 2 illustrates the relaxation of integrality constraints and convex nonlinear constraints.

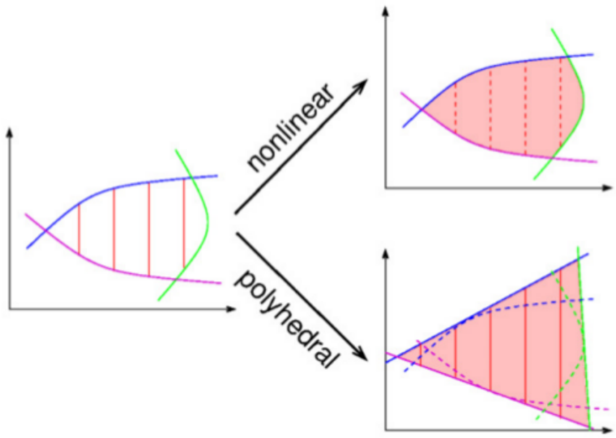

Figure 2. Illustration of the two classes of relaxation

The left image shows the mixed-integer feasible set (the union of the red lines), the top right image shows the nonlinear relaxation obtained by relaxing the integrality constraints (the shaded area is the NLP feasible set), and the bottom right figure shows a polyhedral relaxation (the union of the red lines) as well as its LP relaxation (the shaded area).

Note that an infinite number of possible polyhedral relaxations exists, depending on the choice of the points$x(k) \in conv(X)$, $k = 1, \ldots, K$. If the solution to a relaxation is feasible in (6), then it also solves the MINLP. In general, however, the solution is not feasible in (6), and we must somehow exclude this solution from the relaxation.

2.2. Linearization Techniques

The first step is to convert the non-binary formulation to a binary or $0-1$ formulation. In other words, the integer variable x is replaced by binary$y$. Assuming that each variable has a finite upper boundu_j, the expression for x can be written as:

$$x_j = x_j = \sum_{i=0}^{t_j} 2^i y_{ij} \quad (11)$$

$$y_{ij} = 0,1$$

$$i \ = 0, \ldots, tj$$

Where t_j is the smallest positive integer such that $u_j \leq 2^{t_j+1} - 1$

The second step is to reduce the polynomial $0-1$ program to a linear $0-1$ program by introducing new $0-1$ variables to take the place of cross product terms. While, the power expression of the type y^n(where$y = 0 \; or \; 1$) can simply be replaced by$y$.

Let Q be the set of $0-1$ variables, then every distinct product $\prod_{j\in Q} y_j$ of $0-1$ variables would be replaced by a new $0-1$ variablesy_Q. In order to ensure that $y_Q = 1$ if and only if$\prod_{j\in Q} y_j = 1$, we impose two new constraints:

$$\sum_{j\in Q} y_j + y_Q + q - 1 \geq 0 \quad (12)$$

$$\sum_{j\in Q} y_j - q y_Q \geq 0 \quad (13)$$

$$y_Q = 0 \; or \; 1 \quad (14)$$

Where q denotes the number of elements in Q. the linearized problem may simply be solved by using any standard algorithm, such as Balas' algorithm. However, the new $0-1$ linear program is formulated at significant cost. For every cross product term an extra binary variable has to be added as well as two additional constraints. Therefore the number of variables and constraints will increase drastically even for small nonlinear $0-1$ programs.

3. UTER APPROXIMATION ALGORITHM

In the previous section described a decomposition method for solving problems belonging to the class of MINLP problem. The method which is called Generalized Bender's Decomposition (GBD) method utilizes the mathematical principles of projection, dual-representation (outer-approximation) and relaxation.

The main steps of the outer-approximation, proposed by [14]. They also use these principles, but in a different way. The main difference is that the outer-approximation algorithm exploits the optimal primal information of the sub-problems rather than the dual information to define a mixed-integer linear master program.

The outer-approximation algorithm handles a particular class of MINLP problem which has the following characteristics. The continuous (x) and integer (y) (or discrete valued) variables are separable, nonlinearities only appear in the continuous variables and the nonlinear functions are defined to be convex. Therefore, mathematically this particular class of problem cab stated in the following form:

$$\begin{aligned} &Minimize && \phi = c^T y + F(x) \\ &Subject\ to && f(x) + By \le 0 \end{aligned}$$

$$x \in X \;, y \in Y$$

$$X = \{(x_1, x_2, \dots, x_n) | l_j \le x_j \le u_j\}$$

$$Y = \{(y_1, y_2, \dots, y_n) | l'_j \le y_j \le u'_j \text{ and integer}\}$$

Where the nonlinear functions $F : R^n \to R$ and those in the vector functions $f : R^n \to R$ are assumed to be continuously differentiable and convex on the n-dimensional compact polyhedral convex set X. The first step of the outer-approximation algorithm is similar to GBD method, that is, to select an integer combination $y^i \in Y$. The MINLP problem P_{out} for fixed y^i then reduces to the following NLP sub-problem in x:

$$\begin{aligned} &Minimize && \phi(y^i) = c^T y^i + F(x) \\ &Subject\ to && f(x) \le -By^i \quad [P_{out}(y^i)] \end{aligned}$$

In order to construct a master program as the main component of such an algorithm, we solve the sub problem $[P_{out}(y^i)]$ above for x. suppose that $(\phi(y^i), x^i)$ is the optimal solution (provided the solution exists). Its optimal objective function value $\phi(y^i)$ provides a valid upper bound on the optimal objective ϕ for sub-problem (P_{out}). Based on the solution of the NLP sub- problem, x^i, an approximation to problem (P_{out}) can be constructed [14].

A linear outer-approximation derived at the point x^i for nonlinear function F and f may be expressed with the following relation:

$$F(x) \ge (a^i)^T x - b^i \quad (15)$$

$$f(x) \geq D^i x - d^i \; \forall x \in X$$

Where a^i is the vector of variable coefficients for objective linearization b^i is the constant coefficients for objective linearization d^i is the vector of constant coefficients for integer variables, and D^i is the matrix of variable coefficients for constraint linearization. Because of the assumption that the nonlinear functions are continuously differentiable and convex, the tangential approximation evaluated at x can be expressed as:

$$(a^i)^T x - b^i \equiv F(x^i) + \nabla F(x^i)^T (x - x^i) \tag{16}$$

$$D^i x - d^i \quad \equiv f(x^i) + \nabla f(x^i)^T (x - x^i) \tag{17}$$

$$\forall x \in X$$

Where $\nabla F(x^i)$ is the n-gradient vector and $\nabla f(x^i)$ is the $n \times p$ Jacobian matrix evaluated at given$x^i \in X$.

4. IMPROVEMENT HEURISTICS

Several heuristics to search for a feasible solution of a MINLP have been proposed recently. They all make use of LP, MILP, and NLP solvers to solve problems easier than the MINLP to obtain a feasible point. Some of these heuristics may completely ignore the objective function and focus on finding a feasible solution. They may use the solution of the relaxation at any node in the branch-and-bound as a starting point and try to help out for the lack of focus on the objective function of the MINLP.

Improvement heuristics start with a given feasible point x^* of the MINLP and try to find a better point. Two well-known heuristics for searching a better solution in the neighborhood of a known solution have been adapted from MILP to MINLP.

4.1. Local Branching

Local branching is a heuristic for MINLPs, where all integer variables are binary, that is, $x_i \in \{0,1\}, \forall i \in I$. It was first introduced in the context of MILP by [11] and generalizes readily to convex MINLPs. By describing local branching for convex MINLPs and then discuss an extension to nonconvex MINLPs. The main idea behind local branching is to use a generic MILP solver at a tactical level that is controlled at a strategic level by a simple external branching framework. Assume that given a feasible incumbent x^* of (5), and consider the following disjunction (generalized branching) for a fixed constant k ∈Z:

$$\|x_I - x_I^*\|_1 \leq k \text{ (Left branch) or}$$

$$\|x_I - x_I^*\|_1 \geq k + 1 \text{ (Right branch)} \tag{18}$$

This disjunction corresponds to the Hamming distance of x_I fromx_I^*, and the left branch can also be interpreted as a l_1 trust region around the incumbent. In the case of binary variables, we can rewrite (18) as two linear constraints:

$$\sum_{i \in I; x_i^* = 0} x_i + \sum_{i \in I; x_i^* = 0} (1 - x_i) \leq k \qquad \text{(left)}$$

Or

$$\sum_{i\in I;x_i^*=0} x_i + \sum_{i\in I;x_i^*=0} (1-x_i) \geq k+1 \text{ (right)}$$

The left branch is constructed in such a way that it is much easier to solve than (5), typically by choosing k [15] and [16]. By solving the left branch using any of the deterministic methods.

4.2. Relaxation-Induced Neighborhood Search (RINS)

In the RINS [17] heuristic, one searches for better solutions in the neighborhood of an already known solution, much like local branching. However, instead of imposing a distance constraint (18) to determine a neighborhood, variables are fixed to certain values. The variables to be fixed are selected on the basis of the solution of the relaxation x' , and the already known incumbent x^*. For all $i \in I$ the variables are fixed to $x_i^*, x_i' = x_i^*$. If the fixing reduces the problem size considerably, then it can be solved by calling the solver again [18] extend this idea from MILP to the NLP-based branch-and-bound algorithm for convex MINLP. Once they fix the integer variables as above, they solve the smaller MINLP using the LP/NLP-BB algorithm. They show that the LP/NLP-BB algorithm is much faster on the smaller problems than is the NLP-based branch and-bound algorithm.

5. PROPOSED ALGORITHM FOR SOLVING MINLPS

In the Settlement Algorithm, there are two types of methods used namely exact and heuristic methods. The exact method is used in the nonlinear program settlement. The heuristic method is used for the determination of the starting point used in completing the nonlinear program. Then the next heuristic method is used to obtain integer completion.

The outline of the basic framework of the algorithm are:

Step 1 : Obtain a heuristic completion point for completion of nonlinear programs.

Step 2 : Solve the nonlinear program of relaxation results from the MINLP problem by using the starting point obtained in step 1. If the optimum value of this relaxation problem has been obtained a worthy settlement. then Stop. The solution to MINLP has been obtained. If not, go to step 3.

Step 3 : Obtain a heuristic method to obtain a worthy settlement solution.

Step 4 : Check the results obtained from step 3 if it can still be further, advanced to step 5 otherwise Stop.

Step 5 : Go back to step 3.

6. CONCLUSION

The method for solving MINLP problems is in three major classes. The first class is formed by the exact method class (or also called the deterministic method), provided the problem meet certain conditions such as convexity, will be guaranteed to reach (convergent) at the optimum settlement point or may indicate that there is no integer completion. In general, this exact method has something in common, that is, doing a thorough tree search with rules capable of limiting exploration of sub-trees. The second class is the heuristic method. This class does not guarantee that at the time of the process of stopping at the perspective point of outcome is an optimal value. The third class is the hybrid method.

There are two types of hybrids, namely hybrids between heuristic methods and other heuristic methods and hybrid methods between exact methods and heuristic methods. This study discusses the second hybrid method. In this research resulted in the settlement algorithm to solve the MINLP problem with the exact and heuristic hybrid approach.

In the given algorithmic stages, we will determine the search method and the variables in finding a reasonable solution. The hybrid method aims to gain computational performance or conceptual simplification, potentially at the cost of accuracy or precision. For further research, we will do experiments in large-scale data settlement and obtain testing, and validation the algorithm developed.

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