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Determining Membership Function of Fuzzy Logic Using Genetic Algorithm based on Max-Min Composition

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Abstract

Fuzzy logic is determined as a set of mathematical principles for knowledge representation based on degrees of membership rather than on crisp membership of classical binary logic. Membership function determines the performance of fuzzy logic as it relates to represent fuzzy set in a computer. In this paper we have proposed a method by using Max-Min Composition and Genetic Algorithm for determining membership function of fuzzy logic. Membership function used in this study is triangular and trapezoidal MF, which is the simplest form of Membership Function. Max-Min Composition is a method to calculate a Fuzzy Relation in Fuzzy Logic. A relation is a mathematical description of a situation where certain elements of sets are related to one another in some way. Using Max-Min Composition, we have a degree of relationship between the elements. Relations on fuzzy stating how strong the relationship between elements in fuzzy. The higher degree of relation level means the stronger the relationship between elements. Genetic algorithm is a heuristic search algorithm based on the idea of natural selection that occurs in the process of evolution and genetic operations. This algorithm perform an intelligent search for a solution and have a broad spectrum of possible sollution. We can combine the max-min composition method to get an idea of the strength of the relationship between elements. Based on the strength of that relationship we can determine the interval membership function by using a genetic algorithm.
Keywords: Genetic Algorithm, Alpha Parameter, Arithmetic Crossover, Heuristic Crossover

1. Introduction

Fuzzy logic is determined as a set of mathematical principles for knowledge representation based on degrees of membership rather than on crisp membership of classical binary logic [12].

Fuzzy systems are a part of soft computing that works on the discipline of vagueness and gives results in an interpretable manner. Fuzzy system makes use of fuzzy set theory, fuzzy reasoning and inference mechanism so that such systems can be employed in various applications. In classical set theory an object can either a member of a given set or not while fuzzy set theory allows an object to belong to a set with a certain degree. Fuzzy system models fuzzy boundaries of linguistic terms by introducing gradual membership [6].

Membership function determines the performance of fuzzy logic as it relates to represent fuzzy set in a computer[1].

In every day content most of the problems involve imprecise concept. To handle the imprecise concept, the conventional method of set theory and numbers are insufficient and need to be extended to some other concepts. Fuzzy concept is one of the concepts for this purpose. A relation is a mathematical description of a situation where certain elements of sets are related to one another in some way. Fuzzy relations are significant concepts in fuzzy theory and have been widely used in many fields such as fuzzy clustering, fuzzy control and uncertainty reasoning. They also play an important role in fuzzy diagnosis and fuzzy modeling. When fuzzy relations are used in practice, how to estimate and compare them is a significant problem. Uncertainty measurements of fuzzy relations have been done by some researchers[4].

Fuzzy relation in different product space can be combined with each other by the operation called "Composition". There are many composition methods in use, e.g. max-product method, max-average method and max-min method. But max-min composition method is best known in fuzzy logic applications[14]. Using Max-Min Composition, we have a degree of relationship between the elements. Relations on fuzzy stating how strong the relationship between elements in fuzzy. The higher degree of relation level means the stronger the relationship between elements. Genetic algorithms (GAs) are a class of evolutionary algorithms made popular by John Holland and his colleagues during the 1970s [2]. Genetic Algorithm is

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a searching method used for choosing the best solution of the different problems, based on the mechanism of natural selection. That is, from the initial population, through several evolutionary steps, a set of new more appropriate solutions are achieved that led to the global optimal solution [5]. This algorithm perform an intelligent search for a solution and have a broad spectrum of possible sollution. We can combine the max-min composition method to get an idea of the strength of the relationship between elements. Based on the strength of that relationship we can determine the interval membership function by using a genetic algorithm.

2. Working of Fuzzy Logic

Fuzzy logic consists of three main structure: fuzzification, inference engine, and defuzzification [10]. The fuzzy logic structure can be seen in Figure 1.

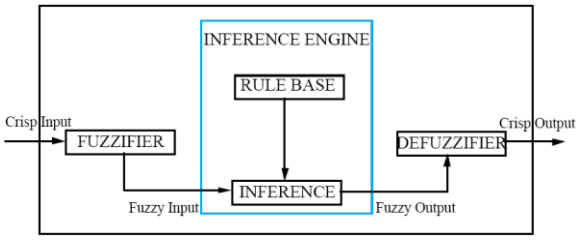

Fig 1. Fuzzy Logic Process [10]

Fuzzy logic continues to grow, at this time we have known type 2 fuzzy logic. Type-2 fuzzy sets (T2 FSs), originally introduced by Zadeh [7], provide additional design degrees of freedom in Mamdani and TSK fuzzy logic systems (FLSs), which can be very useful when such systems are used in situations where lots of uncertainties are present [9]. The resulting type-2 fuzzy logic systems (T2 FLS) have the potential to provide better performance than a type-1 (T1) Fuzzy Logic System [8].

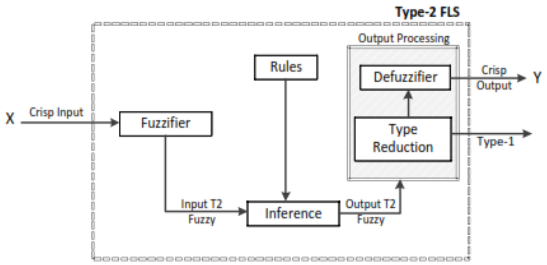

Fig 2. Type 2 Fuzzy Logic Process [3]

The differences between type 2 and type 1 is the type 2 have phases type reduction.

3. Composition of Relation in Fuzzy Logic

Let $X, Y \subseteq R$ be Universal Sets then [14];

$$R = \{((x,y), \mu_R(x,y)) \mid (x,y) \in X \times Y\}$$

Is Called a Fuzzy Relation i $X \times Y \subseteq R$

Or X and Y are two universal sets, the fuzzy relation $R(x,y)$ is given as

$$R(x,y) = \left\{ \frac{\mu_R(x,y)}{(x,y)} \mid (x,y) \in X \times Y \right\} \quad (1)$$

Fuzzy Relations are often presented in the form of two dimensional tables. A $m \times n$ matrix represents a contented way of entering the fuzzy relation R.

$$R = \begin{array}{c} \\ x_1 \\ \vdots \\ x_m \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & \cdots & y_n \end{array} \\ \begin{bmatrix} \mu_R(x_1,y_1) & \cdots & \mu_R(x_1,y_n) \\ \vdots & \ddots & \vdots \\ \mu_R(x_m,y_1) & \cdots & \mu_R(x_m,y_n) \end{bmatrix} \end{array} \quad (2)$$

Fuzzy relation in different product space can be combined with each other by the operation called "Composition". There are many composition methods in use , e.g. max-product method, max-average method and max-min method. But max-min composition method is best known in fuzzy logic applications.

4. Max-Min Composition

Let X, Y and Z be universal sets and let R be a relation that relates elements from X to Y, i.e [4].

$R = \{((x,y), \mu_R(x,y))\}$ $x \in X, y \in Y, R \subset X \times Y$

And

$Q = \{((y,z), \mu_Q(y,z))\}$ $y \in Y, z \in Z, Q \subset Y \times Z$

Then S will be a relation that relates elements in X that R contains to the elements in Z that Q contains, i.e.

$S = \{((x,z), \mu_S(x,z))\}$ $x \in X, z \in Z, S \subset X \times Z$

Max-Min Composition is then defined as

$$\mu_S(x,z) = \max_{y \in Y}(\min(\mu_R(x,y), \mu_Q(y,z))) \quad (3)$$

5. Genetic Algorithm

Genetic Algorithm (GA) is adaptive heuristic based on ideas of natural selection and genetics. Genetic algorithm is one of the most known categories of evolutionary algorithm. A GA works with a number of solutions which collectively is known as population in each iteration which is chosen randomly. These are adaptive heuristic search algorithms postulated on the evolutionary ideas of natural selection and genetic. The basic concept of these evolutionary algorithms is to stimulate process in natural system necessary for

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evolution. GA's are used for numerical and computational optimization and based on study the evolutionary aspects of models of social systems. The GA performs a balanced search on various nodes and there is a need to retain population diversity exploration so that any important information cannot be lost because there is a great need to focus on fit portions of the population.

The simplest form of genetic algorithm involves three types of operators: selection, crossover, and mutation [11].

Selection This operator selects chromosomes in the population for reproduction. The fitter the chromosome, the more times it is likely to be selected to reproduce.

Crossover This operator randomly chooses a locus and exchanges the subsequences before and after that locus between two chromosomes to create two offspring. For example, the strings 10000100 and 11111111 could be crossed over after the third locus in each to produce the two offspring 10011111 and 11100100. The crossover operator roughly mimics biological recombination between two single−chromosome (haploid) organisms.

Mutation This operator randomly flips some of the bits in a chromosome. For example, the string 00000100 might be mutated in its second position to yield 01000100. Mutation can occur at each bit position in a string with some probability, usually very small (e.g., 0.001).

The process of genetic algroritma can be seen in Figure 3.

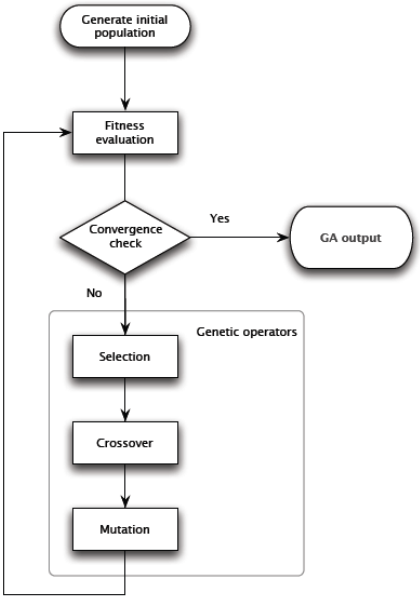

Fig. 3 Flowchart of Genetic Algorithm [13]

5. Determining Membership Function Using Max-Min Composition and Genetic Algorithm

Assume that we have Fuzzy Logic System with the following rules.

1. If X is X1 and Y is Y1 then Z is Z1
2. If X is X1 and Y is Y1 then Z is Z3
3. If X is X1 and Y is Y1 then Z is Z4
4. If X is X1 and Y is Y2 then Z is Z1
5. If X is X1 and Y is Y2 then Z is Z2
6. If X is X1 and Y is Y2 then Z is Z3
7. If X is X1 and Y is Y4 then Z is Z1
8. If X is X1 and Y is Y4 then Z is Z2
9. If X is X1 and Y is Y4 then Z is Z3
10. .....

That can be represents in relation matrix from *x to y* and from *y to z*.

Assume that matrix relation from *x to y* and *y to z* is as follows.

$$R_1 = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccccc} y_1 & y_2 & y_3 & y_4 & y_5 \end{array} \\ \begin{bmatrix} 0.1 & 0.2 & 0 & 1 & 0.7 \\ 0.3 & 0.5 & 0 & 0.2 & 1 \\ 0.8 & 0 & 1 & 0.4 & 0.3 \end{bmatrix} \end{array}$$

$$R_2 = \begin{array}{c} \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{array} \begin{array}{c} \begin{array}{cccc} z_1 & z_2 & z_3 & z_4 \end{array} \\ \begin{bmatrix} 0.9 & 0 & 0.3 & 0.4 \\ 0.2 & 1 & 0.8 & 0 \\ 0.8 & 0 & 0.7 & 1 \\ 0.4 & 0.2 & 0.3 & 0 \\ 0 & 1 & 0 & 0.8 \end{bmatrix} \end{array}$$

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And Max-Min Composition from R1º R2 (x, z) can we calculate.

$$\mu_{R_1 \circ R_2}(x_1, z_1) = \max(\min(0.1, 0.9), \min(0.2, 0.2), \min(0, 0.8), \min(1, 0.4), \min(0.7, 0))$$
$$= \max(0.1, 0.2, 0, 0.4, 0) = 0.4$$

Similarly we can determine the grades of membership for all pairs, as we obtain:

$$(x_i, z_i), i = 1, 2, 3, j = 1, \ldots\ldots 4$$

$$R_1 o R_2 = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} z_1 & z_2 & z_3 & z_4 \end{array} \\ \begin{bmatrix} 0.4 & 0.7 & 0.3 & 0.7 \\ 0.3 & 1 & 0.5 & 0.8 \\ 0.8 & 0.3 & 0.7 & 1 \end{bmatrix} \end{array}$$

Based on the existing relationship, suppose that we specify that the only relationship that has degree> = 0.7 which we will use. Data set that can we get from Max-Min Composition are data set that has degree relation: 0.7 and 0.8. There are 5 data set. Suppose a data set from a relation of X to Z can be seen in Table 1.

Table 1. Data Set of Relation X to Z

Data Number	X	Z
1	1	2
2	2	3
3	4	5
4	6	7
5	7	8

The interval of Triangular membership Function that be determined by genetic algorithm can be seen in Figure 4.

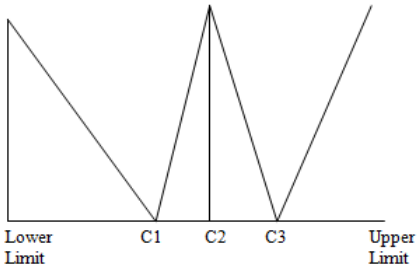

Fig. 4 Triangular MF

The process of the genetic algorithm are as follows.

1. Encode the parameter set (C1, C2, C3) in the form of Bit Strings
   Bit strings are created with random assignment of 1 and 0 at different bit locations. We start with an initial population of five strings (Table 2, column 2). The Strings are 18 bits in length. The first 6 bits encode the C1, the middle 6 bits encode the C2, and the last 6 bits encode the C3.
2. Column 2, 4 and 6, shows the decimal equivalent of their binary coding. These binary values for C1, C2, and C3 are the mapped into values relevan to the problem with these equation.
   $$Ci = C\min + \frac{b}{2^L - 1}(CMaxi - C\min i)$$
   (Here, Cmin i= -2 and Cmax i = 5, using these values we compute C1, C2, and C3)
3. These values shown in Table 2 column 8, 9, 10, 11, and 12 are the values computed using the equation.
   Z' = C1X + C2 + C3
   Using the values 0f C1, C2, and C3 from column 3, 5, and 7, respectively for different value of x as given in Table 1.
4. These computed values for the Z' are compared with the Z as the correct values, and the square of the errors in estimated for each string.
   This summation is subtracted from a large number, that can be determined according to the problem (2200 in this problem) to convert the problem into maximization problem.

Table 2. First Iteration Using GA

String	C1 (Bin)	C1	C2 (Bin)	C2	C3 (Bin)	C3	Y1	Y2	Y3	Y4	Y5
000111 010100 010101	7	-1.22	20	0.22	21	0.33	-0.67	-1.89	-4.33	-6.77	-7.99
010010 001100 100001	18	-0.00003	12	-0.667	33	1.67	1.00297	1.00294	1.00288	1.00282	1.00279
010101 101010 011001	21	0.33	42	2.67	25	0.78	3.78	4.11	4.77	5.43	5.76
100100 001001 100011	36	2	9	-1	35	1.89	2.89	4.89	8.89	12.89	14.89
001001 001111 101000	9	-0.99	15	-0.33	40	2.44	1.12	0.13	-1.85	-3.83	-4.82

5. We can compute the fitness of each strings.
   Fitness 1 = $2200 - \sum(zi' - zi)^2 = 23.7775$
   Fitness 2 = $2200 - \sum(zi' - zi)^2 = 1800..576$
   Fitness 3 = $2200 - \sum(zi' - zi)^2 = 2198.678$
   Fitness 4 = $2200 - \sum(zi' - zi)^2 = 1821.698$
   Fitness 5 = $2200 - \sum(zi' - zi)^2 = 1026.938$
   Best Fitness = 2198.678
6. Determine the value of C1, C2, and C3 from the best fitness.
   According to the best fitness, we can see the value of C1 is 21, C2 is 25 and C3is 42.
7. We can go to the next generation or the next iteration if we want to get the better fitness than the fitness from generation 1.
   At this step we can go to the step of genetic algorithm, such as: selection, crossover, and mutation
8. Process will stop if we feel that we get the best fitness or the fitness value seems not to be changed to next generation.

6. Discussion

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Max-Min Composition and Genetic Algorithm can be used for determining membership function of fuzzy logic. Using Max-Min Composition, we have a degree of relationship between the elements. Relations on fuzzy stating how strong the relationship between elements in fuzzy. The higher degree of relation level means the stronger the relationship between elements. Based on the strength of that relationship we can determine the interval membership function by using a genetic algorithm. In the process, we can encode the parameter set (C1, C2, C3) in the form of Bit Strings. The Strings are 18 bits in length. The first 6 bits encode the C1, the middle 6 bits encode the C2, and the last 6 bits encode the C3 and the calculate a binary of C1, C2, C3, and also the value of Z'. Next process is calculate the fitness value. According to the best fitness, we can calculate the value of C1, C2, and C3.

7. Conclusions

The conclusion that can be drawn from this study are as follows.

1. Using Max-Min Composition, we have a degree of relationship between the elements
2. Genetic Algorithm can be used in determining membership function of fuzzy logic.
3. Value of C1, C2, and C3 from the best fitness can be used as an interval of membership function in fuzzy logic.

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