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Rumor Propagation - an Operational Research Approach by Computational and Information Theory

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Abstract Rumors are a kind of information having impact that has a impact on social life and economies, which spread quickly and widely, especially, via internet. Recently, the spread of information affects our daily lives due to the increasing number of social media users. Rumours are defined in social area which are delivered by gossips, fake news, marketing, social media all the way even to revolutions. In this paper, we study the dynamics of a rumour propagation model with a numerical approach. By using an algorithmic technique with an error analysis, the validity of the numerical technique is described. We investigate the model with this numerical approach to explain the dynamics of rumour propagation. Moreover, by numerical simulations the efficiency of the technique is shown. Finally, the results are displayed and discussed with the help of figures and tables. The paper ends with a conclusion and an outlook of future studies.

Keywords Operational research · information dynamics · differential equations · numerical methods · error bounds · simulation

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1 Introduction

Mathematical models have a great importance in many areas such as engineering, economics, finance, biology, physics, and social science. Epidemiological models are subject of investigation in biology, e.g., in the study of disease dynamics that present the mechanism of disease transmission. Various epidemiological models of disease propagation can be represented so-called SIR models [1]. Here, S stands for the currently 'Susceptible' section of the population, I stands for the currently 'Infected' section of the population in our focus, and R denotes the section of the population who are currently 'Removed' from the state of infectibility. Social and biological interactions have always been the subject of attention for scientists. For instance, well-known oral presently diseases, chronic social instability, anxiety, etc., have been analyzed and modeled. As some interesting studies on infectious diseases and vaccination, and on related phenomena, we refer to [2]-[3] and references given therein.

In this paper, we consider the dynamics of a rumour propagation model with a numerical approach. In previous decades, the propagation of rumours in a population has become a research topic of increasing interest in the fields of computer sciences, mathematics, physics, engineering and social sciences. Important pioneering OR contributions to an understanding of rumour propagation were provided by S.C. Belen, Ch. Pearce, et al. [4]-[7]. Motivations for these investigations come from different perspectives such as social sciences, economics, informatics, defense and military related inquiries. We all remember the dynamics of the so called Arabic Spring and Chinese whispers [8]-[9] with the vast role of news' and opinions' spread played by social media and social networks [10]-[13]. In many countries, there are children games demonstrating how the meaning of words can change along a chain of participants who pass the word from one to the next person [14]. In the examples of business and political parties, it is quite apparent that information spread is a major phenomenon and a key tool. Here, the sectors of emerging marketing industries and of defense against so-called fake news are just two of many examples [15].

Recently, there has been a rapidly increased interest in studying various forms of social interaction both due to the availability of computational power and observable datasets from our modern virtual on-line social networks. Spread, mutation and transformation of information are being explored. Based on these remarks, mathematical models for the spread of rumours are inspired by compartmental computational epidemic models, where the population is divided into different compartments depending on their status addressed [4].

Regarding rumour propagation, similarly with the SIR model, the population is generally divided into three groups: 'Ignorant', ones $u_1(t)$, in number, are individuals who do not know that rumour, 'Spreaders', in number, $u_2(t)$, are individuals who know and spread the rumour), and 'Stiflers', ones of them, $u_3(t)$, are individuals who know the rumour but do not spread it. We note that for analytic inquiries we all these quantities to be real numbers; especially, when these numerical figures are high, this simplification seems to be meaningful and permitted. These types of models are hard to resolve and often arises in the contexts of numerical approaches which have been investigated by many authors. Therefore, homotopy analysis method, collocation methods, stochastic Galerkin method, WENO numerical scheme, etc., have been studied and applied to obtain approximate solutions of the SIR models [16]-[21].

This paper is organized as follows. In Section 2, the mathematical model and its evolution is introduced. In Section 3, fundamental relations of the present technique are described. Our method is explained in Section 4. Accuracy of the technique has been investigated in Section 5. Our algorithm is shown in Section 6. In Section 7, the numerical technique is applied on our model, and the results are displayed by figures and tables. There is a brief conclusion with an outlook on future studies in Section 8.

2 Model

SIR model was constructed in 1927 by W. O. Kermack and A. G. McKendrick [22]. This model is a fixed population with three compartments: susceptible, infected, and recovered where $S(t)$ is used to represent the number of individuals not yet infected with the disease at time t, or those susceptible to the disease, $I(t)$ denotes the number of individuals who have been infected with the disease and are capable of spreading the disease to those in the susceptible category, and $R(t)$ is the compartment used for those individuals who have been infected and then recovered from the disease [23]. Those who are in this category are not able to be infected again or to transmit the infection to others. In this model, Kermack and McKendrick assumed a time-dependent population, i.e., $N(t) = S(t) + I(t) + R(t)$, where $N(t)$ is the population and derived the following system:

$$\begin{cases} \dot{S}(t) &= -\beta S I, \\ \dot{I}(t) &= \beta S I - \gamma I, \\ \dot{R}(t) &= \gamma I. \end{cases} \tag{1}$$

Here, β is considered as the contact or infection rate of the disease, γ represents the mean recovery rate. Similarly, from Eq. (1) we describe a deterministic model for the rumour propagation as

$$\begin{aligned}
\frac{du_1(t)}{dt} &= -\beta u_1(t)u_2(t)/N(t),\\
\frac{du_2(t)}{dt} &= \beta u_1(t)u_2(t)/N(t) - \gamma u_2(t)/N(t),\\
\frac{du_3(t)}{dt} &= \gamma u_2(t)/N(t),
\end{aligned} \tag{2}$$

where ignorant individuals become spreaders at a rate β and then become stiflers at a rate γ, and $N(t) = u_1(t) + u_2(t) + u_3(t)$ is considered to be constant and normalized to 1 [24]-[26]. Moreover, the initial conditions are given as $u_1(0) = \lambda_1$, $u_2(0) = \lambda_2$ and $u_3(0) = \lambda_3$.

3 Fundamental Relations

Our aim is to find approximate solution of our given Eq. (2) in the truncated Laguerre series form

$$u_i(t) \cong u_{i,N}(t) = \sum_{n=0}^{N} a_n L_n(t); \quad i = 1, 2, 3, \ 0 \le t \le b < \infty, \tag{3}$$

where a_n are unknown coefficients and $L_n(t)$ are the Laguerre polynomials for $n = 0, 1, \ldots, N$, and which are defined as

$$L_n(t) = \sum_{r=0}^{n} \frac{(-1)^r}{r!} \binom{n}{r} t^r, \quad n \in N, \quad 0 \le t < \infty. \tag{4}$$

We compose the matrix forms of Eq. (3) to find the matrix representations of each term in the system:

$$[u_i(t)] = \mathbf{L}(t)\mathbf{A}_i, \quad i = 1, 2, 3, \tag{5}$$

where

$$\begin{aligned}
\mathbf{L}(t) &= \left[\, L_0(t)\ L_1(t) \cdots\ L_N(t) \,\right], \quad \text{and}\\
\mathbf{A}_i &= \left[\, a_{i,0}\ a_{i,1} \cdots\ a_{i,N} \,\right]^T, \quad i = 0, 1, \ldots, N.
\end{aligned}$$

Now, we present $\mathbf{L}(t)$ in matrix form as

$$\mathbf{L}(t) = \mathbf{X}(t)\mathbf{H}, \tag{6}$$

where

$$\begin{aligned}
\mathbf{X}(t) &= \left[\, 1\ t \cdots\ t^N \,\right],\\
\mathbf{H} &= \begin{bmatrix}
\frac{(-1)^0}{0!}\binom{0}{0} & \frac{(-1)^0}{0!}\binom{1}{0} & \cdots & \frac{(-1)^0}{0!}\binom{N}{0}\\
0 & \frac{(-1)^1}{1!}\binom{1}{1} & \cdots & \frac{(-1)^1}{1!}\binom{N}{1}\\
\vdots & \vdots & \ddots & \vdots\\
0 & 0 & \cdots & \frac{(-1)^N}{N!}\binom{N}{N}
\end{bmatrix}.
\end{aligned}$$

Then we convert the solution defined by Eq. (5) and its derivatives, for $n = 0, 1, \ldots, N$, to the following matrix forms, using Eq. (6):

$$[u_i(t)] = \mathbf{X}(t)\mathbf{H}\mathbf{A}_i. \tag{7}$$

We also define the matrix relations between $u_i(t)$ and its first-order $u_i^{'}(t)$ as

$$[u_i^{'}(t)] = \mathbf{X}(t)\mathbf{H}\mathbf{B}\mathbf{A}_i, \tag{8}$$

where

$$\mathbf{B} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & N \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}.$$

Furthermore, the matrix relation of nonlinear part in Eq. (2) is defined as

$$[u_1(t)u_2(t)] = \mathbf{L}(t)\mathbf{L}^*(t)\overline{\mathbf{A}}_{2,1} = \mathbf{X}(t)\mathbf{H}\mathbf{X}^*(t)\mathbf{H}^*\overline{\mathbf{A}}_{2,1}, \tag{9}$$

where

$$\mathbf{X}^*(t) = \begin{bmatrix} \mathbf{X}(t) & 0 & \cdots & 0 \\ 0 & \mathbf{X}(t) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{X}(t) \end{bmatrix}, \ \mathbf{H}^* = \begin{bmatrix} \mathbf{H} & 0 & \cdots & 0 \\ 0 & \mathbf{H} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{H} \end{bmatrix},$$
$$\overline{\mathbf{A}}_{2,1} = \begin{bmatrix} a_{1,0}\mathbf{A}_2 \ a_{1,1}\mathbf{A}_2 \ \cdots \ a_{1,N}\mathbf{A}_2 \end{bmatrix}^T.$$

For this purpose, substituting the matrix Eqns. (7), (8) and (9), into Eq. (2) and simplifying, we obtain the following matrix equations:

$$\begin{aligned} [u_1^{'}(t)] &= \mathbf{X}(t)\mathbf{H}\mathbf{B}\mathbf{A}_1 = -\beta\mathbf{X}(t)\mathbf{H}\mathbf{X}^*(t)\mathbf{H}^*\overline{\mathbf{A}}_{2,1}, \\ [u_2^{'}(t)] &= \mathbf{X}(t)\mathbf{H}\mathbf{B}\mathbf{A}_2 = \beta\mathbf{X}(t)\mathbf{H}\mathbf{X}^*(t)\mathbf{H}^*\overline{\mathbf{A}}_{2,1} - \gamma\mathbf{X}(t)\mathbf{H}\mathbf{A}_2, \\ [u_3^{'}(t)] &= \mathbf{X}(t)\mathbf{H}\mathbf{B}\mathbf{A}_3 = \gamma\mathbf{X}(t)\mathbf{H}\mathbf{A}_2, \end{aligned} \tag{10}$$

or

$$\begin{cases} \mathbf{X}(t)\mathbf{H}\mathbf{B}\mathbf{A}_1 + \beta\mathbf{X}(t)\mathbf{H}\mathbf{X}^*(t)\mathbf{H}^*\overline{\mathbf{A}}_{2,1} = [f_1(t)], \\ \mathbf{X}(t)\mathbf{H}\mathbf{B}\mathbf{A}_2 - \beta\mathbf{X}(t)\mathbf{H}\mathbf{X}^*(t)\mathbf{H}^*\overline{\mathbf{A}}_{2,1} + \gamma\mathbf{X}(t)\mathbf{H}\mathbf{A}_2 = [f_2(t)], \\ \mathbf{X}(t)\mathbf{H}\mathbf{B}\mathbf{A}_3 - \gamma\mathbf{X}(t)\mathbf{H}\mathbf{A}_2 = [f_3(t)]. \end{cases}$$

where $f_1(t),\ f_2(t)$ and $f_3(t)$ are continuous functions [27]-[28]. Alternatively,

$$\begin{cases} \mathbf{D}_1(t)\mathbf{A}_1 + \mathbf{E}_1(t)\overline{\mathbf{A}}_{2,1} = [f_1(t)], \\ \mathbf{D}_2(t)\mathbf{A}_2 + \mathbf{E}_2(t)\overline{\mathbf{A}}_{2,1} = [f_2(t)], \\ \mathbf{D}_3(t)\mathbf{A}_3 + \mathbf{D}_4(t)\mathbf{A}_2 = [f_3(t)], \end{cases} \tag{11}$$

where

$$\begin{aligned}\mathbf{D}_1(t) &= \mathbf{X}(t)\mathbf{HB},\\ \mathbf{D}_2(t) &= \mathbf{X}(t)\mathbf{HB} + \gamma\mathbf{X}(t)\mathbf{H},\\ \mathbf{D}_3(t) &= \mathbf{X}(t)\mathbf{HB},\\ \mathbf{D}_4(t) &= -\gamma\mathbf{X}(t)\mathbf{H},\end{aligned}$$

and

$$\begin{aligned}\mathbf{E}_1(t) &= \beta\mathbf{X}(t)\mathbf{HX}^*(t)\mathbf{H}^*,\\ \mathbf{E}_2(t) &= -\beta\mathbf{X}(t)\mathbf{HX}^*(t)\mathbf{H}^*.\end{aligned}$$

Consequently, the fundamental matrix equations of Eq. (11) can be written in the following form:

$$\mathbf{D}_j(t)\mathbf{A}_j + \mathbf{E}_k(t)\overline{\mathbf{A}}_{2,1} = \mathbf{f}(t); \quad j = 1,2,3,4; \quad k = 1,2, \tag{12}$$

where

$$\mathbf{D}_j(t) = \begin{bmatrix} \mathbf{D}_1(t) & 0 & 0 \\ 0 & \mathbf{D}_2(t) & 0 \\ 0 & \mathbf{D}_4(t) & \mathbf{D}_3(t) \end{bmatrix},\ \mathbf{E}_k(t) = \begin{bmatrix} \mathbf{E}_1(t) \\ \mathbf{E}_2(t) \\ 0 \end{bmatrix},\ \mathbf{f}(t) = \begin{bmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \end{bmatrix}.$$

4 Method of Solution

The problem in Eq. (2) with the initial conditions is to be solved over the interval $[0, c_m h]$. We choose c_m from $0 \leq c_1 \leq \cdots \leq c_m \leq b < \infty$. The affiliated collocation points $t_l = 0 + c_m h$ for $m = 0,1,\ldots,N$. So, we define the collocation points as

$$t_l = \frac{b}{N} l, \quad l = 0,1,\ldots,N, \quad \text{and,} \quad h = \frac{l}{N}. \tag{13}$$

The collocation points are replaced into Eq. (12) we obtain the system

$$\mathbf{D}_j(t_l)\mathbf{A}_j + \mathbf{E}_k(t_l)\overline{\mathbf{A}}_{2,1} = \mathbf{f}(t_l), \quad j = 1,2,3,4; \quad k = 1,2; \quad l = 0,1,\ldots,N.$$

Then the fundamental matrix equation is found as

$$\mathbf{WA}^* + \mathbf{V}\overline{\overline{\mathbf{A}}} = \mathbf{F},\ \text{in short: } [\mathbf{W} : \mathbf{V}; \mathbf{F}], \tag{14}$$

where

$$\mathbf{W} = \begin{bmatrix} \mathbf{D}_j(t_0) & 0 & \cdots & 0 \\ 0 & \mathbf{D}_j(t_1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{D}_j(t_N) \end{bmatrix},\ \mathbf{A}^* = \begin{bmatrix} \mathbf{A}_j \\ \mathbf{A}_j \\ \vdots \\ \mathbf{A}_j \end{bmatrix},$$

$$\mathbf{V} = \begin{bmatrix} \mathbf{E}_k(t_0) & 0 & \cdots & 0 \\ 0 & \mathbf{E}_k(t_1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{E}_k(t_N) \end{bmatrix},\ \overline{\overline{\mathbf{A}}} = \begin{bmatrix} \overline{\mathbf{A}} \\ \overline{\mathbf{A}} \\ \vdots \\ \overline{\mathbf{A}} \end{bmatrix}.$$

We find matrix representation of the initial conditions by following a similar procedure as

$$\begin{aligned}
[u_1(0)] &= \mathbf{X}(0)\mathbf{H}\mathbf{A}_1 = [\lambda_1], \\
[u_2(0)] &= \mathbf{X}(0)\mathbf{H}\mathbf{A}_2 = [\lambda_2], \\
[u_3(0)] &= \mathbf{X}(0)\mathbf{H}\mathbf{A}_3 = [\lambda_3].
\end{aligned} \tag{15}$$

Now, replacing the matrices (15) into the last rows of the part $\mathbf{W}$ in Eq. (14), we have the new augmented matrix as $[\tilde{\mathbf{W}} : \tilde{\mathbf{V}}; \tilde{\mathbf{F}}]$. Hence solving the system, from Eq. (3), an approximate solution of Eq. (2) with initial conditions is obtained in the form of Eq. (3) [29]-[30].

5 Accuracy

In this section, we check the accuracy of the method. We consider the error estimate for Laguerre approximation. First we define $e_{i,N}(t)$, the error function by using $u_i(t)$ and $u_{i,N}(t)$ exact and approximate solutions, respectively [31]:

$$e_{i,N}(t) = u_i(t) - u_{i,N}(t).$$

We describe Eq. (2) in the form

$$u_i^{'}(t) = g_i(t, u_1, ..., u_i); \quad i = 1, 2, 3. \tag{16}$$

Now, by the help of Eq. (16) we define the residual error function as

$$R_{i,N}(t) = u_{i,N}(t) - g_{i,N}(t, u_1, ..., u_i), \tag{17}$$

where we have the exact solution whenever we have $R_{i,N}(t) = 0$. On the other hand, taking Eq. (16) and Eq. (17) yields

$$\begin{aligned}
e_{i,N}^{'}(t) - g_i(t, e_{1,N}, ..., e_{i,N}) - u_i^{'}(t) + g_i(t, u_1, ..., u_i) \\
+ u_{i,N}^{'}(t) - g_i(t, u_{1,N}, ..., u_{i,N}) + R_{i,N}(t) = 0,
\end{aligned}$$

with the homogeneous conditions $u_1 = 0$, $u_2 = 0$, and $u_3 = 0$, where we have the approximation from $e_{i,N,M}(t)$ to $e_{i,N}(t)$ for $M \geq N$. The approximate solutions $u_{1,N}(t)$, $u_{2,N}(t)$, and $u_{3,N}(t)$ of Eq. (2), and their first-order are considered and substituted into Eq. (2). Then we obtain approximate results for $t = t_r \in [0, R]$, $R = 0, 1, \ldots$.

$$\begin{aligned}
E_{1,N}(t_r) &= \left| u_{1,N}(t_r) - u_1^{'}(t_r) - \beta u_1(t_r) u_2(t_r)/N(t_r) \right| \cong 0, \\
E_{2,N}(t_r) &= \left| u_{2,N}(t_r) - u_2^{'}(t_r) + \beta u_1(t_r) u_2(t_r)/N(t_r) - \gamma u_2(t_r)/N(t_r) \right| \cong 0, \\
E_{3,N}(t_r) &= \left| u_{3,N}(t_r) - u_3^{'}(t_r) + \gamma u_2(t_r)/N(t_r) \right| \cong 0,
\end{aligned}$$

where $E_{i,N}(t_r)$, $i = 1, 2, 3$, are absolute error functions, and $E_{i,N}(t_r) \leq 10^{-p_{\alpha,\sigma}}$ for p any positive integer. Then we have the approximate results whenever N is chosen large enough.

6 Algorithm

In this section, we consider the *Steps* of algorithm for the present technique:

Data: β, γ constants in Eq. (2).
Result: $u_{1,N}(t)$, $u_{2,N}(t)$, and $u_{3,N}(t)$ approximate solutions.

S0. Truncation is chosen as $m \leq N$ for $m \in \mathbb{N}$,
S1. Construction of all the matrices,
S2. Replacement of the fundamental matrix equation,
S3. Apply the collocation points, $t_l = \frac{b}{N}l, \quad l = 0, 1, \ldots, N, \quad$ and, $\quad h = \frac{l}{N}$, to the fundamental matrix equation in S2.
S4. Computation of the augmented matrix $[\mathbf{W} : \mathbf{V}; \mathbf{F}]$ by Gauss-Elimination,
S5. Construction of the initial conditions in matrix forms $[u_1(0)]$, $[u_2(0)]$, and $[u_3(0)]$,
S6. Replacement of the initial conditions in matrix forms in *S5.* to the augmented matrix in *S4.* then we get $[\tilde{\mathbf{W}} : \tilde{\mathbf{V}}; \tilde{\mathbf{F}}]$,
S7. Solution of the system in *S6.* and replacement in the truncated Laguerre series form in Eq. (3).
S8. Stop.

A systematic approach has been taken using the algorithm to find all approximate solutions of the unknowns in Eq. (2). Then we settle the accuracy of the solutions in order to investigate efficient results for describing the dynamics of the model.

7 Numerical Experiments

Let us consider $\beta = 0.01$ and $\gamma = 0.02$ in Eq. (2) [17]-[18], [21]; the initial conditions are given as:

- Initial population of $u_1(t)$, who are ignorant, $u_1(0) = \lambda_1 = 25$,
- Initial population of $u_2(t)$, who are spreaders, is $u_2(0) = \lambda_2 = 15$, and
- Initial population of $u_3(t)$, who are stiflers, is $u_3(0) = \lambda_3 = 10$.

Table 1: Absolute errors for $N = 8$ and $N = 10$; comparison for $u_1(t)$.

t	$E_{1,8}$	$E_{1,10}$
0.0	0.00000	0.00000
0.1	0.00000	0.00000
0.2	0.00000	0.00000
0.3	0.00000	0.00000
0.4	0.40000E-7	0.10300E-7
0.5	0.16000E-6	0.32000E-7
0.6	0.49000E-5	0.20600E-7
0.7	0.11700E-5	0.11700E-7
0.8	0.25000E-5	0.52000E-7
0.9	0.48800E-5	0.12800E-7
1.0	0.88100E-5	0.40000E-7

Table 2: Absolute errors for $N = 8$ and $N = 10$; comparison for $u_2(t)$.

t	$E_{2,8}$	$E_{2,10}$
0.0	0.00000	0.00000
0.1	0.2429990E-3	0.00000
0.2	0.9719960E-3	0.00000
0.3	0.2186989E-3	0.00000
0.4	0.3887982E-3	0.15000E-7
0.5	0.6074971E-4	0.21000E-7
0.6	0.8747960E-4	0.36200E-7
0.7	0.1190694E-4	0.14500E-7
0.8	0.1555193E-4	0.25500E-7
0.9	0.1968293E-4	0.19000E-7
1.0	0.2429994E-4	0.14900E-7

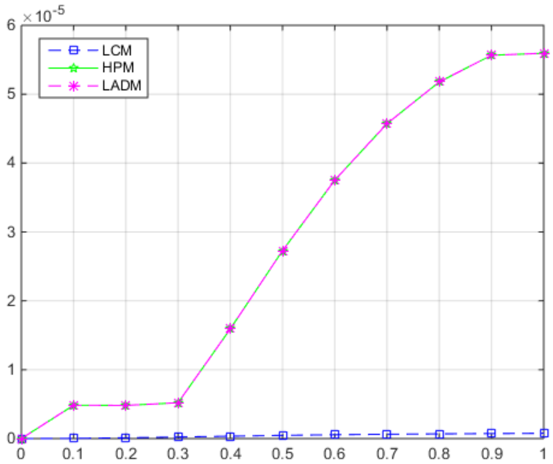

Fig. 1: Comparison of the error function for Laguerre Collocation Method (LCM), Homotopy Perturbation Method (HPM) and Laplace-Adomian decomposition method (LADM) for $u_3(t)$, $N = 4$.

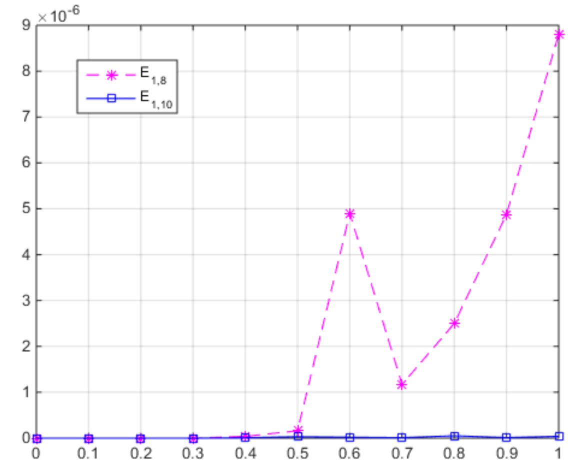

Fig. 2: Comparison of the absolute errors for $u_1(t)$, $N = 8$ and $N = 10$.

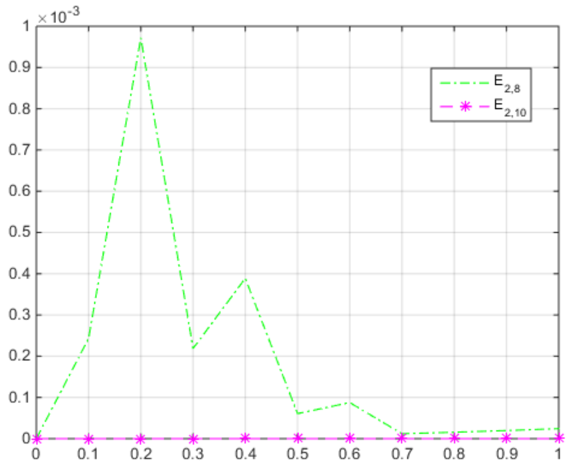

Fig. 3: Comparison of the absolute errors for $u_2(t)$, $N = 8$ and $N = 10$.

8 Conclusion

In this study, we have reached the rumour propagation model on optimization and real-life decision making based on the dynamics which we modeled [32]-[33]. The role of the parameters in the dynamics show us the interaction of spread of rumours by ignorant, spreaders, and stiflers. This social phenomenon of rumour has been reached by a novel numerical technique from Operational Research and Information theory.

We have applied Laguerre collocation method on the ODEs system by considering continuous time and initial conditions. We have shown the efficiency and accuracy of the approximate solutions by the present technique with an example. Tables and figures have been demonstrated that the approximation results mean a valuable contribution as an alternative method in understanding information spread, and in economic and societal practice.

We will apply and investigate needed parameter estimation based one real-world data in the future applications. Furthermore, we could turn to more explicit time dependence in our differential equations, and regime-switching phenomena could be addressed as well. In forthcoming studies, our investigations can become employed and extended towards challenges of marketing, spread of new fashions, ideologies, but also of democracy and its novel trends.

Conflict of interest

The authors declare that they have no conflict of interest.

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