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Rumour propagation: an operational research approach by computational and information theory

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Abstract
Rumours are a kind of information that has a impact on social life and economies, which spread quickly and widely, especially, via internet. Recently, the spread of information affects our daily lives due to the increasing number of social media users. Rumours are defined in social areas which are delivered by gossips, fake news, marketing, social media all the way even to revolutions. In this paper, we study the dynamics of a rumour propagation model with a numerical approach. By using an algorithmic technique with an error analysis, the validity of the numerical technique is described. We investigate the model with this numerical approach to explain the dynamics of rumour propagation. Besides, we explain sensitivity analyses of the model of parameters. Then by numerical simulations efficiency of the technique is shown. Finally, the results are displayed and discussed with the help of figures and tables. The paper ends with a conclusion and an outlook to future studies.

Keywords Operational research · Information dynamics · Differential equations · Numerical methods · Error bounds · Sensitivity analysis · Simulation

1 Introduction

Mathematical models have a great importance in many areas such as engineering, economics, finance, biology, physics, and social science. Epidemiological models are subject of investigation in biology, e.g., in the study of disease dynamics that present the mechanism of disease transmission. Various epidemiological models of disease propagation can be represented so-called SIR models (Murray 2003). Here, S stands for the currently 'Susceptible' section of the population, I stands for the currently 'Infected' section of the population in our focus, and R denotes the section of the

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population who are currently 'Removed' from the state of infectibility. These type of epidemic dynamics explain the transmission of infectious diseases from one individual to another by using the variable function of t. The compartmental model is a dynamic can be expressed by the compartments in which oscillations may appear. So, the model can explain mostly spread of an endemic disease with a short period of time. Its well-known example was measles, contagious infectious disease, in the UK prior to the introduction of a vaccine in 1968. Therefore, the susceptible infectious recovered (SIR) dynamics is known as the commonly used epidemic spreading model (Compartmental 2020; Wang et al. 2020; Li and Zhang 2017). Social and biological interactions have always been the subject of attention for scientists. For instance, well-known oral presently diseases, chronic social instability, anxiety, etc., have been analyzed and modeled. As some interesting studies on infectious diseases and vaccination, and on related phenomena, we refer to Bhattacharyya et al. (2019) and Enright and Kao (2018) and references given therein.

Motivation of the work is the study of rumour propagation with the idea of infection networks of people, particularly, understanding spread of information. In this paper, we consider the dynamics of a rumour propagation model with a numerical approach. In previous decades, the propagation of rumours in a population has become a research topic of increasing interest in the fields of computer sciences, mathematics, physics, engineering and social sciences. Important pioneering OR contributions to an understanding of rumour propagation were provided by Belen et al. (2011), Belen et al. (2008), Akgümüş and Weber (2011) and Gomaa et al. (2016). Motivations for these investigations come from different perspectives such as social sciences, economics, informatics, defense and military related inquiries. We all remember the dynamics of exciting so called Arabic Spring and also of so-called Chinese whispers Social media (2020) and Chinese whispers (2020), with the vast role of news' and opinions' spread played by social media and social networks Khondker (2011),Howard and Hussain (2013), Wolfsfeld et al. (2013) and Stepan and Linz (2013). In many countries, there are children games demonstrating how the meaning of words can change along a chain of participants who pass the word from one to the next person (Lyytimäki et al. 2014). In the examples of business and political parties, it is quite apparent that information spread is a major phenomenon and a key tool. Here, the sectors of emerging marketing industries and of defense against so-called fake news are just two of many examples (Majchrzak et al. 2019).

Recently, there has been a rapidly increased interest in studying various forms of social interaction both due to the availability of computational power and observable datasets from our modern virtual on-line social networks. Spread, mutation and transformation of information are being explored. Based on these remarks, mathematical models for the spread of rumours are inspired by compartmental computational epidemic models, where the population is divided into different compartments depending on their status addressed (Belen et al. 2011).

Regarding rumour propagation, similarly with the SIR model, the population is generally divided into three groups: 'Ignorant' ones, $u_1(t)$ in number, are individuals who do not know that rumour, 'Spreaders', in number: $u_2(t)$, are individuals who know and spread the rumour), and 'Stiflers', $u_3(t)$ ones of them, are individuals who know the rumour but do not spread it. We note that for analytic inquiries we need all these quantities to be real numbers. Especially, when these numerical figures are high, this simplification seems to be meaningful and permitted. These types of models are hard to resolve and often arise in the contexts of numerical approaches which have been investigated by many authors. Therefore, homotopy analysis method, collocation methods, stochastic Galerkin method, WENO numerical scheme, etc., have been studied and applied to obtain approximate solutions of the SIR models (Biazar 2006; Rafei et al. 2007; Doğan and Akın 2012; Ibrahim et al. 2018; Seçer et al. 2018; Harman and Johnston 2016).

This paper is organized as follows. In Sect. 2, the mathematical model and its evolution is introduced. In Sect. 3, the present technique is described. Accuracy of the technique has been investigated and the algorithm has been presented in Sect. 4. In Sect. 5, a numerical technique is applied on our model, and the results are displayed by figures and tables. There is a brief discussion on achievements, limitations and implications of the study in Sect. 6. A final conclusion as an outlook on future studies is given in Sect. 7.

2 Model

SIR model was constructed in 1927 by Kermack and McKendrick (1927). This model is a fixed population with three compartments: susceptible, infected, and recovered where $S(t)$ is used to represent the number of individuals not yet infected with the disease at time t, or those susceptible to the disease, $I(t)$ denotes the number of individuals who have been infected with the disease and are capable of spreading the disease to those in the susceptible category, and $R(t)$ is the compartment used for those individuals who have been infected and then recovered from the disease (Breda et al. 2012). Those who are in this category are not able to be infected again or to transmit the infection to others. In this model, Kermack and McKendrick assumed a time-dependent population, i.e., $N(t) = S(t) + I(t) + R(t)$, where $N(t)$ is the population and derived the following system:

$$\begin{cases} \dot{S}(t) &= -\beta S I, \\ \dot{I}(t) &= \beta S I - \gamma I, \\ \dot{R}(t) &= \gamma I. \end{cases} \tag{1}$$

Here, β is considered as the contact or infection rate of the disease, γ represents the mean recovery rate. Here, our aim is to describe our main model that is a deterministic model for the rumour propagation model with respect to Eq. (1), that we define as

$$\begin{aligned} \frac{du_1(t)}{dt} &= -\beta u_1(t) u_2(t)/N(t), \\ \frac{du_2(t)}{dt} &= \beta u_1(t) u_2(t)/N(t) - \gamma u_2(t)/N(t), \\ \frac{du_3(t)}{dt} &= \gamma u_2(t)/N(t). \end{aligned} \tag{2}$$

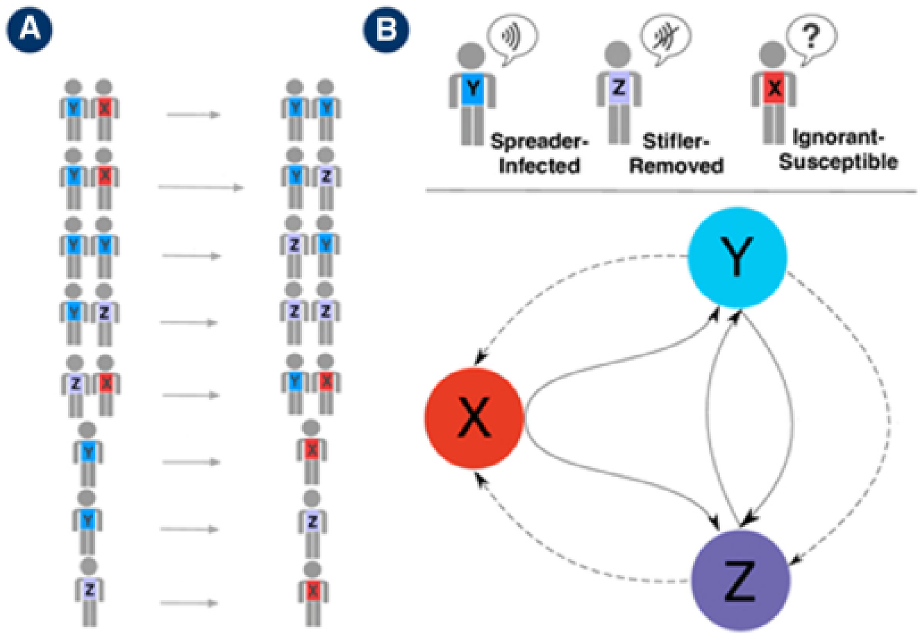

Fig. 1 A systematic description of the model

Here, ignorant individuals become spreaders at a rate β and then become stiflers at a rate γ, and $N(t) = u_1(t) + u_2(t) + u_3(t)$ is considered to be constant and normalized to 1, without loss of generality Ragagnin (2016), Gebert et al. (2007) and Temocin and Weber (2014). Moreover, the initial conditions are given as $u_1(0) = \lambda_1$, $u_2(0) = \lambda_2$ and $u_3(0) = \lambda_3$.

The model explains us the propagation of rumours in a population. This model is of increasing interest recently in many fields such as social sciences, economy, informatics, military, and so on. Mainly, we consider a simple model which has been important to understand the essential mechanisms in the contact processes. The social networks of customers may also be exploited by companies to promote their products; these investigations affect the economy directly and inspire operational research studies. On the other hand, the extreme boom of online social networks has been investigated in many fields as well. These concepts have an important impact on rumour propagation as it is seen at Fig. 1 as well (Ferraz de Arruda et al. 2018). Due to this reason, we deal with the deterministic model for the rumour propagation model. In order to explore the dynamics, we pay regard to a numerical technique which is a collocation method based on Laguerre polynomials.

3 Method

3.1 Fundamental relations

Here, we show the fundamental relations of our technique which help us to construct the numerical method and figure out the dynamics efficiently. Firstly, we display an approximation scheme with a related series form. Therefore, we know that our aim is to find approximate solution of our given Eq. (2) in the truncated Laguerre series form of

$$u_i(t) \cong u_{i,N}(t) = \sum_{n=0}^{N} a_n L_n(t); \quad i = 1, 2, 3, \ 0 \le t \le b < \infty, \tag{3}$$

where a_n are unknown coefficients, and $L_n(t)$ are the Laguerre polynomials for $n = 0, 1, \ldots, N$, which are defined as

$$L_n(t) = \sum_{r=0}^{n} \frac{(-1)^r}{r!} \binom{n}{r} t^r, \quad n \in N, \quad 0 \le t < \infty. \tag{4}$$

We compose the matrix forms of Eq. (3) to find the matrix representations of each term in the system:

$$[u_i(t)] = \mathbf{L}(t)\mathbf{A}_i, \quad i = 1, 2, 3, \tag{5}$$

where

$$\mathbf{L}(t) = \left[L_0(t) \ L_1(t) \cdots L_N(t) \right], \quad \text{and}$$
$$\mathbf{A}_i = \left[a_{i,0} \ a_{i,1} \cdots a_{i,N} \right]^T, \quad i = 0, 1, \ldots, N. \tag{6}$$

Now, we present $\mathbf{L}(t)$ in matrix form as

$$\mathbf{L}(t) = \mathbf{X}(t)\mathbf{H}, \tag{7}$$

where

$$\mathbf{X}(t) = \left[1 \ t \cdots t^N \right],$$
$$\mathbf{H} = \begin{bmatrix} \frac{(-1)^0}{0!}\binom{0}{0} & \frac{(-1)^0}{0!}\binom{1}{0} & \cdots & \frac{(-1)^0}{0!}\binom{N}{0} \\ 0 & \frac{(-1)^1}{1!}\binom{1}{1} & \cdots & \frac{(-1)^1}{1!}\binom{N}{1} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{(-1)^N}{N!}\binom{N}{N} \end{bmatrix}. \tag{8}$$

Then we convert the solution defined by Eq. (5) and its derivatives, for $n = 0, 1, \ldots, N$, to the following matrix forms, using Eq. (7):

$$[u_i(t)] = \mathbf{X}(t)\mathbf{H}\mathbf{A}_i. \tag{9}$$

We also define the matrix relations between $u_i(t)$ and its first-order derivative $u_i'(t)$ as

$$[u_i'(t)] = \mathbf{X}(t)\mathbf{H}\mathbf{B}\mathbf{A}_i, \tag{10}$$

where

$$\mathbf{B} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & N \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}. \tag{11}$$

Furthermore, the matrix relation of nonlinear part in Eq. (2) is defined as

$$[u_1(t)u_2(t)] = \mathbf{L}(t)\mathbf{L}^*(t)\overline{\mathbf{A}}_{2,1} = \mathbf{X}(t)\mathbf{H}\mathbf{X}^*(t)\mathbf{H}^*\overline{\mathbf{A}}_{2,1}, \tag{12}$$

where

$$\mathbf{X}^*(t) = \begin{bmatrix} \mathbf{X}(t) & 0 & \cdots & 0 \\ 0 & \mathbf{X}(t) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{X}(t) \end{bmatrix}, \ \mathbf{H}^* = \begin{bmatrix} \mathbf{H} & 0 & \cdots & 0 \\ 0 & \mathbf{H} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{H} \end{bmatrix},$$
$$\overline{\mathbf{A}}_{2,1} = \left[a_{1,0}\mathbf{A}_2 \ a_{1,1}\mathbf{A}_2 \ \cdots \ a_{1,N}\mathbf{A}_2 \right]^T. \tag{13}$$

For this purpose, substituting the matrix Eqs. (9), (10) and (12), into Eq. (2) and simplifying, we obtain the following matrix equations:

$$\begin{aligned} [u_1'(t)] &= \mathbf{X}(t)\mathbf{H}\mathbf{B}\mathbf{A}_1 = -\beta\mathbf{X}(t)\mathbf{H}\mathbf{X}^*(t)\mathbf{H}^*\overline{\mathbf{A}}_{2,1}, \\ [u_2'(t)] &= \mathbf{X}(t)\mathbf{H}\mathbf{B}\mathbf{A}_2 = \beta\mathbf{X}(t)\mathbf{H}\mathbf{X}^*(t)\mathbf{H}^*\overline{\mathbf{A}}_{2,1} - \gamma\mathbf{X}(t)\mathbf{H}\mathbf{A}_2, \\ [u_3'(t)] &= \mathbf{X}(t)\mathbf{H}\mathbf{B}\mathbf{A}_3 = \gamma\mathbf{X}(t)\mathbf{H}\mathbf{A}_2, \end{aligned} \tag{14}$$

or

$$\begin{cases} \mathbf{X}(t)\mathbf{H}\mathbf{B}\mathbf{A}_1 + \beta\mathbf{X}(t)\mathbf{H}\mathbf{X}^*(t)\mathbf{H}^*\overline{\mathbf{A}}_{2,1} = [f_1(t)], \\ \mathbf{X}(t)\mathbf{H}\mathbf{B}\mathbf{A}_2 - \beta\mathbf{X}(t)\mathbf{H}\mathbf{X}^*(t)\mathbf{H}^*\overline{\mathbf{A}}_{2,1} + \gamma\mathbf{X}(t)\mathbf{H}\mathbf{A}_2 = [f_2(t)], \\ \mathbf{X}(t)\mathbf{H}\mathbf{B}\mathbf{A}_3 - \gamma\mathbf{X}(t)\mathbf{H}\mathbf{A}_2 = [f_3(t)]. \end{cases} \tag{15}$$

Here, $f_1(t)$, $f_2(t)$ and $f_3(t)$ are continuous functions (Gürbüz and Sezer 2016; Çetin et al. 2018). Alternatively,

$$\begin{cases} \mathbf{D}_1(t)\mathbf{A}_1 + \mathbf{E}_1(t)\overline{\mathbf{A}}_{2,1} = [f_1(t)], \\ \mathbf{D}_2(t)\mathbf{A}_2 + \mathbf{E}_2(t)\overline{\mathbf{A}}_{2,1} = [f_2(t)], \\ \mathbf{D}_3(t)\mathbf{A}_3 + \mathbf{D}_4(t)\mathbf{A}_2 = [f_3(t)], \end{cases} \tag{16}$$

where

$$\begin{aligned} \mathbf{D}_1(t) &= \mathbf{X}(t)\mathbf{HB}, \\ \mathbf{D}_2(t) &= \mathbf{X}(t)\mathbf{HB} + \gamma \mathbf{X}(t)\mathbf{H}, \\ \mathbf{D}_3(t) &= \mathbf{X}(t)\mathbf{HB}, \\ \mathbf{D}_4(t) &= -\gamma \mathbf{X}(t)\mathbf{H}, \end{aligned} \tag{17}$$

and

$$\begin{aligned} \mathbf{E}_1(t) &= \beta \mathbf{X}(t)\mathbf{H}\mathbf{X}^*(t)\mathbf{H}^*, \\ \mathbf{E}_2(t) &= -\beta \mathbf{X}(t)\mathbf{H}\mathbf{X}^*(t)\mathbf{H}^*. \end{aligned} \tag{18}$$

Consequently, the fundamental matrix equations of Eq. (16) can be written in the following form:

$$\mathbf{D}_j(t)\mathbf{A}_j + \mathbf{E}_k(t)\overline{\mathbf{A}}_{2,1} = \mathbf{f}(t); \quad j = 1, 2, 3, 4; \quad k = 1, 2, \tag{19}$$

where

$$\mathbf{D}_j(t) = \begin{bmatrix} \mathbf{D}_1(t) & 0 & 0 \\ 0 & \mathbf{D}_2(t) & 0 \\ 0 & \mathbf{D}_4(t) & \mathbf{D}_3(t) \end{bmatrix}, \ \mathbf{E}_k(t) = \begin{bmatrix} \mathbf{E}_1(t) \\ \mathbf{E}_2(t) \\ 0 \end{bmatrix}, \ \mathbf{f}(t) = \begin{bmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \end{bmatrix}. \tag{20}$$

3.2 Method of solution

The problem in Eq. (2) with the initial conditions is to be solved over the interval $[0, c_m h]$. We choose c_m from $0 \leq c_1 \leq \cdots \leq c_m \leq b < \infty$. The affiliated collocation points are $t_l = 0 + c_m h$ for $m = 0, 1, \ldots, N$. So, we define the collocation points as

$$t_l = \frac{b}{N} l, \quad l = 0, 1, \ldots, N, \quad \text{and,} \quad h = \frac{l}{N}. \tag{21}$$

When the collocation points are inserted into Eq. (19), we obtain the system

$$\mathbf{D}_j(t_l)\mathbf{A}_j + \mathbf{E}_k(t_l)\overline{\mathbf{A}}_{2,1} = \mathbf{f}(t_l), \quad j = 1, 2, 3, 4; \quad k = 1, 2; \quad l = 0, 1, \ldots, N. \tag{22}$$

Then the fundamental matrix equation is found as

$$\mathbf{W}\mathbf{A}^* + \mathbf{V}\overline{\overline{\mathbf{A}}} = \mathbf{F}, \ \text{in short: } [\mathbf{W} : \mathbf{V}; \mathbf{F}], \tag{23}$$

where

$$\mathbf{W} = \begin{bmatrix} \mathbf{D}_j(t_0) & 0 & \cdots & 0 \\ 0 & \mathbf{D}_j(t_1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{D}_j(t_N) \end{bmatrix}, \ \mathbf{A}^* = \begin{bmatrix} \mathbf{A}_j \\ \mathbf{A}_j \\ \vdots \\ \mathbf{A}_j \end{bmatrix},$$
$$\mathbf{V} = \begin{bmatrix} \mathbf{E}_k(t_0) & 0 & \cdots & 0 \\ 0 & \mathbf{E}_k(t_1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{E}_k(t_N) \end{bmatrix}, \ \overline{\overline{\mathbf{A}}} = \begin{bmatrix} \overline{\mathbf{A}} \\ \overline{\mathbf{A}} \\ \vdots \\ \overline{\mathbf{A}} \end{bmatrix}. \tag{24}$$

We find a matrix representation of the initial conditions by following a similar procedure as

$$\begin{aligned} [u_1(0)] &= \mathbf{X}(0)\mathbf{H}\mathbf{A}_1 = [\lambda_1], \\ [u_2(0)] &= \mathbf{X}(0)\mathbf{H}\mathbf{A}_2 = [\lambda_2], \\ [u_3(0)] &= \mathbf{X}(0)\mathbf{H}\mathbf{A}_3 = [\lambda_3]. \end{aligned} \tag{25}$$

Now, replacing the matrices (25) into the last rows of the component matrix $\mathbf{W}$ in Eq. (23), we have a new augmented matrix as $[\tilde{\mathbf{W}} : \tilde{\mathbf{V}}; \tilde{\mathbf{F}}]$. Hence solving the system, from the ansatz of Eq. (3), an approximate solution of Eq. (2) with initial conditions is obtained in the form of Eq. (3) (Gürbüz and Sezer 2017a, b).

4 Accuracy

In this section, we check the accuracy of the method. We consider the error estimate for Laguerre approximation. First we define $e_{i,N}(t)$, the error function, by using $u_i(t)$ and $u_{i,N}(t)$ approximate solutions (Bani-Ahmad et al. 2016):

$$e_{i,N}(t) = u_i(t) - u_{i,N}(t). \tag{26}$$

We describe Eq. (2) in the form

$$u_i^{'}(t) = g_i(t, u_1, ..., u_i); \quad i = 1, 2, 3. \tag{27}$$

Now, by the help of Eq. (27) we define the residual error function as

$$R_{i,N}(t) = u_{i,N}(t) - g_{i,N}(t, u_1, ..., u_i), \tag{28}$$

where we have a better approximate solution whenever we have $R_{i,N}(t) \cong 0$. We analyse the error function to display that it is gained by using the residual function.

On the other hand, taking Eq. (27) and Eq. (28) yields

$$\begin{aligned} e'_{i,N}(t) - g_i(t, e_{1,N}, ..., e_{i,N}) - u'_i(t) + g_i(t, u_1, ..., u_i) \\ + u'_{i,N}(t) - g_i(t, u_{1,N}, ..., u_{i,N}) + R_{i,N}(t) = 0, \end{aligned} \tag{29}$$

with the homogeneous conditions $u_1 = 0$, $u_2 = 0$, and $u_3 = 0$, where we have the approximation from $e_{i,N,M}(t)$ to $e_{i,N}(t)$ for $M \geq N$. The approximate solutions $u_{1,N}(t)$, $u_{2,N}(t)$, and $u_{3,N}(t)$ of Eq. (2), and their first-order are considered and substituted into Eq. (2). Then we obtain approximate error results for $t = t_r \in [0, R]$, $R = 0, 1, \ldots$.

$$\begin{aligned} E_{1,N}(t_r) &= \left| u_{1,N}(t_r) - u'_1(t_r) - \beta u_1(t_r) u_2(t_r)/N(t_r) \right| \cong 0, \\ E_{2,N}(t_r) &= \left| u_{2,N}(t_r) - u'_2(t_r) + \beta u_1(t_r) u_2(t_r)/N(t_r) - \gamma u_2(t_r)/N(t_r) \right| \cong 0, \\ E_{3,N}(t_r) &= \left| u_{3,N}(t_r) - u'_3(t_r) + \gamma u_2(t_r)/N(t_r) \right| \cong 0, \end{aligned} \tag{30}$$

where $E_{i,N}(t_r)$, $i = 1, 2, 3$, are error functions, and $E_{i,N}(t_r) \leq 10^{-p_{\alpha,\sigma}}$ for $p_{\alpha,\sigma}$ being any positive integer. Then we got the approximate results whenever N is chosen large enough.

Our numerical technique is a specific version of the famous collocation methods (Gürbüz and Sezer 2020, 2018; Gürbüz 2019a, b). The essence of this method is to adapt the scheme of collocation combining it with the Laguerre polynomials which provides a remarkable accuracy. In this section, we show a general scheme for displaying the accuracy of the technique.

4.1 Algorithm

In this section, we consider the *Steps* of algorithm for the present technique:

Data: β, γ: constants in Eq. (2).
Result: $u_{1,N}(t)$, $u_{2,N}(t)$ and $u_{3,N}(t)$: approximate solutions.

S0. Truncation is chosen as $m \leq N$ for $m \in \mathbb{N}$,
S1. Construction of all the matrices,
S2. Replacement of the fundamental matrix equation,
S3. Apply the collocation points (colloc. pts.), $t_l = \frac{b}{N} l$, $l = 0, 1, \ldots, N$, and $h = \frac{l}{N}$, to the fundamental matrix equation in *S2*.
S4. Computation of the augmented matrix $[\mathbf{W} : \mathbf{V}; \mathbf{F}]$ by Gauss elimination,
S5. Construction of the initial conditions (ICs) in matrix forms $[u_1(0)]$, $[u_2(0)]$, and $[u_3(0)]$,
S6. Replacement of the initial conditions in matrix forms in *S5*. to the augmented matrix in *S4*. Then we get $[\tilde{\mathbf{W}} : \tilde{\mathbf{V}}; \tilde{\mathbf{F}}]$,
S7. Solution of the system in *S6*. and replacement in the truncated Laguerre series form in Eq. (3).
S8. Stop.

We establish a powerful algorithmic approach for calculating an approximation procedure and to investigate the dynamics behind rumour propagation. This is of great importance on population framework and has made a considerable impact on socio-economic phenomena.

Algorithms uniquely associate rigidity and completeness. The computer programs run can be designed by mathematical algorithms to accelerate the process of calculations. Data are driven, modelled, and constructive and supportive results or output can be obtained very efficiently. The approximate solutions can be reached conveniently with these novel factors.

A systematic approach has been taken by using our algorithm to find all approximate solutions of the unknowns in Eq. (2). Then we settle the accuracy of the solutions in order to investigate efficient results for describing the dynamics of the model. A comprehensive interpretation of the algorithm has been introduced below with the help of a flowchart. This flowchart about the algorithm shows in Fig. 2 that we apply our technology on the dynamics model within a framework of all the details. In this flowchart, we describe the processes with regard to the inputs and the algorithm in general strategy. This comprehensive approach to our design of the technique reflects the computational perspective to this OR research work.

4.2 Sensitivity analyses

An analysis of the system of nonlinear ODEs remains challenging. In this section, we consider a *Sensitivity Analysis* of our model. Sensitivity analysis provides a valuable perception for the initial conditions and the most important parameters in models. Moreover, it supports the accuracy as well as identifies the controlling factors in models. For instance, when some parameters are perturbed in a linear population dynamics, then an estimation of the asymptotic state of the population can be presented by the sensitivity analysis. It may emphasize which parameters must be estimated accurately and which ones just roughly. A sensitivity analysis also aids for determining key processes in models, and to determine which others, while captivating, contribute to the model for its comparatively particular findings.

Here, we consider an iterative process by parameterizing the nonlinear map:

$$\begin{aligned} \mathbf{s}(t+1, \mathbf{p}) &= \mathbf{h}(\mathbf{s}(t, \mathbf{p}), \mathbf{p}), \\ \mathbf{s}(0) &= \boldsymbol{\lambda}, \end{aligned} \tag{31}$$

where $\mathbf{s} \in \mathbb{R}^N$ is the vector of variables. Besides, $\boldsymbol{\lambda} \in \mathbb{R}^M$ is the vector of initial conditions and $\mathbf{p} \in \mathbb{R}^K$ is defined as the vector of parameters. On the other hand, the right side of the Eq. (31) is the nonlinear term which contains linear and nonlinear Leslie and Lefkovitch matrices and their classes of maps. Basically, an iterative process is considered and the nonlinear map is parameterized with the help of Leslie and Lefkovitch matrices. These matrices are popular matrices in population ecology which describe the growth of populations and their distribution in dynamics.

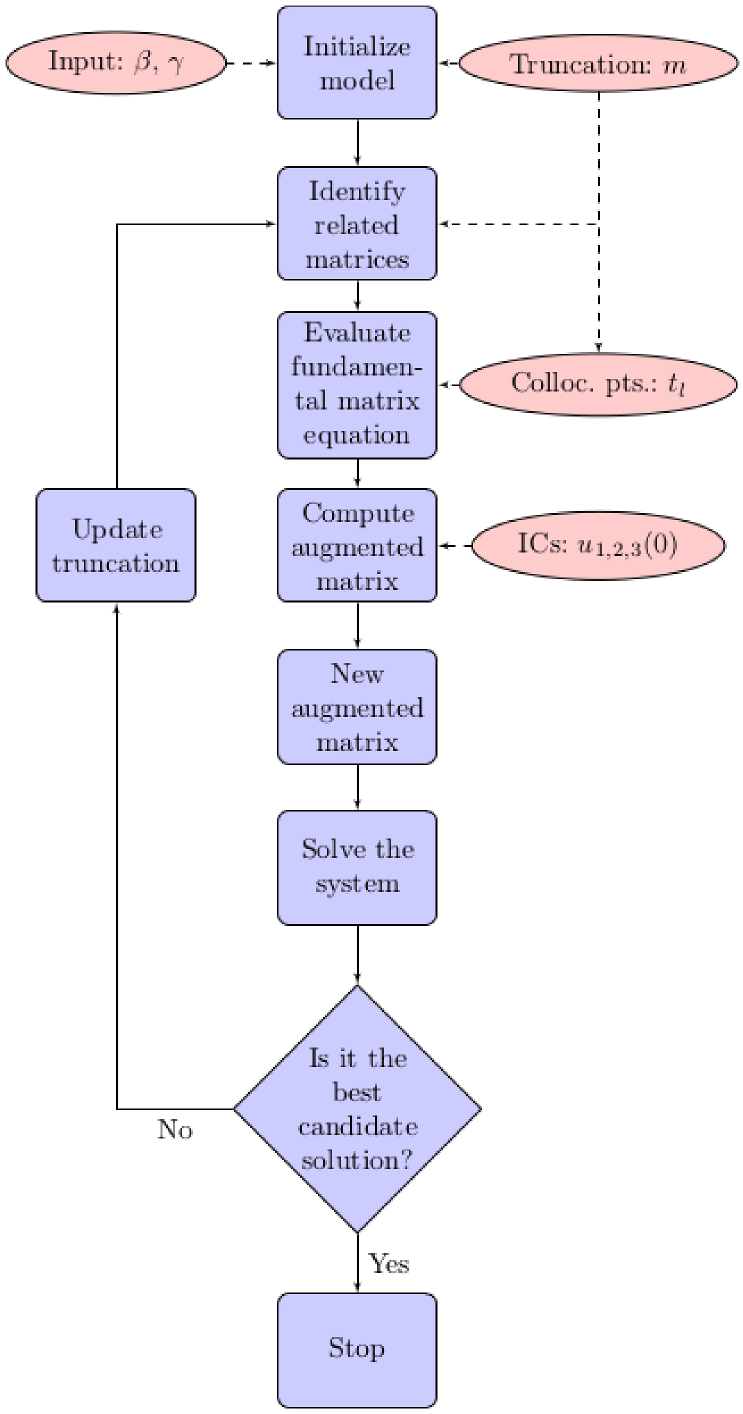

Fig. 2 Flowchart for the algorithm

Now, we consider an equilibrium solution $\mathbf{s}^*(\mathbf{p})$ and we write:

$$\begin{aligned} \mathbf{h}(\mathbf{s}^*(\mathbf{p}), \mathbf{p}) &= \mathbf{s}^*(\mathbf{p}), \\ s_i(t+1, \mathbf{p}) &= h_i(\mathbf{s}(t, \mathbf{p}), \mathbf{p}), \\ s_i(0) &= \lambda_i, \ i = 1, ..., N. \end{aligned} \tag{32}$$

Then we put $S_{i,k}$ as the sensitivity of the i-th variable with regard to the k-th parameter:

$$S_{i,k} = \frac{\partial s_i}{p_k}. \tag{33}$$

This is called as the forward sensitivity analysis approach. Therefore, we can provide sensitivity analyses for all variables in the equations which are of the form in Eq. (32) Tavener et al. (2011), Cushing (1988), Leslie (1945)and Lefkovitch (1965). The sensitivity analysis approach leads to a detailed investigation of the model and its specific results with respect to the initial conditions and the parameters. These results have been shown in the numerical part of our study.

4.3 Algorithm

In this section, we consider an algorithm for the sensitivity analysis is given as:

Data: Define number of equations and number of parameters.
Result: Sensitivity results for dynamical model with related to its parameters.

S0. ODE model (imap=0), choose to compute solutions only,
S1. Define number of parameters and equations: $kdim$, $sdim$, $svec$ and $pvec$,
S2. Map solution parameters for coding the parameters,
S3. Construct nonlinear map.
S4. Define the quantity of interest: qoi,
S5. Define a combination of parameters together with the initial conditions and parameter values: s_0 and p_0,
S6. Set plot data,

– *Write equations*,
– *Write user defined quantity of interest*,
– *Write user defined parameters*,
– *Write initial conditions and parameter values*,
– *Write plot data*,

S8. Stop (Tavener et al. 2011).

We apply a sensitivity analysis which is a typical forward sensitivity analysis and it helps us to state the population variables in our model, Eq.(2) with related to the parameters. Then we center of attraction on the calculation of the sensitivities of transient states of rumour propagation. In this sense, we explain the steps of these calculations with the help of an algorithm above and a flow chart below in Fig. 3.

In our study, we consider the sensitivity analyses of the model with related to the parameters and an approximation is applied in order to investigate the behaviour of the dynamic. In some cases, uncertainty analysis of the model can be described. However, even if uncertainty analysis and sensitivity analysis are closely related which are dissimilar research topics. Uncertainty analysis aims to evaluate the contributions of the inputs while uncertainty outcomes. However, in several works are included these analyses together with the elasticities of the models in order to investigate the

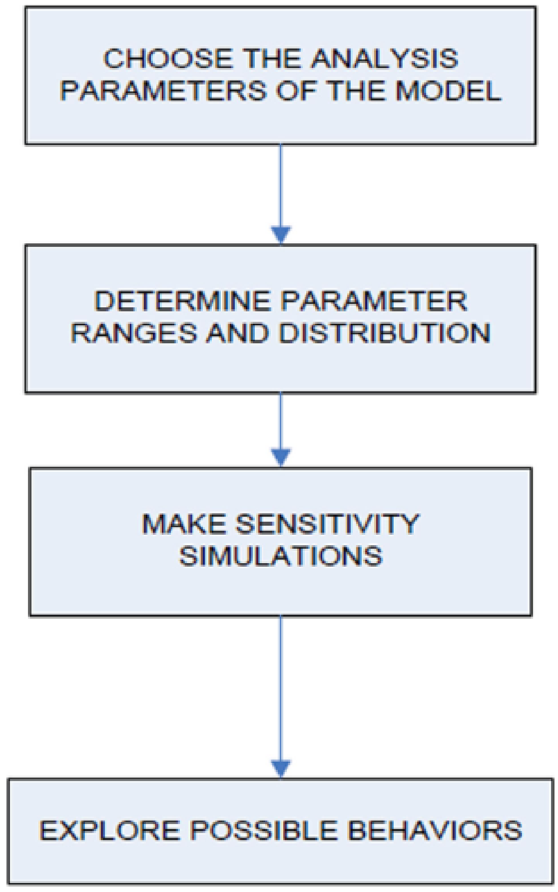

Fig. 3 A flowchart for the sensitivity analyses in our usage (Hekimoğlu and Barlas 2010)

convenience of the model with an accurate relation between input and output data (Tavener et al. 2011; Loucks and van Beek 2017; Caswell 2009).

5 Numerical experiments

In this section, we implement the numerical simulations and our findings with regard to our model and the technique designed. All calculations have been performed by Matlab and Maple softwares. We use the algorithm which has been given previously. Besides, sensitivity analyses of the results have been conducted with respect to our findings. We have used several packages for the implementations which give us an idea about the dynamic model (Tavener et al. 2011).

Let us choose, e.g. $\beta = 0.01$ and $\gamma = 0.02$ in Eq. (2) (Rafei et al. 2007; Doğan and Akın 2012; Harman and Johnston 2016); the initial conditions are given as:

- Initial population of $u_1(t)$, who are ignorants, $u_1(0) = \lambda_1 = 25$,
- Initial population of $u_2(t)$, who are spreaders, is $u_2(0) = \lambda_2 = 15$, and
- Initial population of $u_3(t)$, who are stiflers, is $u_3(0) = \lambda_3 = 10$.

In Table 1, we can see that the error function is increasing by time while it approximates better whenever we have larger truncation limit. This makes us understand an idea that the numerical approximation to the ignorant individuals are perfectly visible at the beginning of the time period. From Table 1 we can realise that, approximately, we

Table 1 Error functions for $N = 8$ and $N = 10$; comparison for $u_1(t)$

t	$E_{1,8}$	$E_{1,10}$
0.0	0.00000	0.00000
0.1	0.00000	0.00000
0.2	0.00000	0.00000
0.3	0.00000	0.00000
0.4	0.40000E−7	0.10300E−7
0.5	0.16000E−6	0.32000E−7
0.6	0.49000E−5	0.20600E−7
0.7	0.11700E−5	0.11700E−7
0.8	0.25000E−5	0.52000E−7
0.9	0.48800E−5	0.12800E−7
1.0	0.88100E−5	0.40000E−7

Table 2 Error functions for $N = 8$ and $N = 10$; comparison for $u_2(t)$

t	$E_{2,8}$	$E_{2,10}$
0.0	0.00000	0.00000
0.1	0.2429990E−3	0.00000
0.2	0.9719960E−3	0.00000
0.3	0.2186989E−3	0.00000
0.4	0.3887982E−3	0.15000E−7
0.5	0.6074971E−4	0.21000E−7
0.6	0.8747960E−4	0.36200E−7
0.7	0.1190694E−4	0.14500E−7
0.8	0.1555193E−4	0.25500E−7
0.9	0.1968293E−4	0.19000E−7
1.0	0.2429994E−4	0.14900E−7

have an enough calculation for the ignorant individuals till $t = 0.4$. After this specific value of t, the error function is increasing and the function $u_1(t)$ is of result with less accuracy.

We also see a similar scenario in Table 2. Here, we can understand that the number of spreaders is assessed by better approximated results since the approximation giving us more efficient results in earlier t time values. However, after $t = 0.4$, we have the error function is increasing along our approximation with our truncation limit as $N = 10$. Besides, we can show that the approximation to the function $u_2(t)$ which represents the number of spreaders has efficient results in between specific time period of t between $0 - 0.5$. Around the middle of our investigation, at first the error is increasing, while in the end of our observation it is decreasing. It also be seen in Table 2 that we have a better approximation whenever we have larger values for N.

If we compare some different techniques for the same truncation limit, $N = 4$, our technique gives us better results, which can be seen in Fig. 4. In this specific dynamics, we have better approximation results for our numerical technique, Laguerre Colloca-

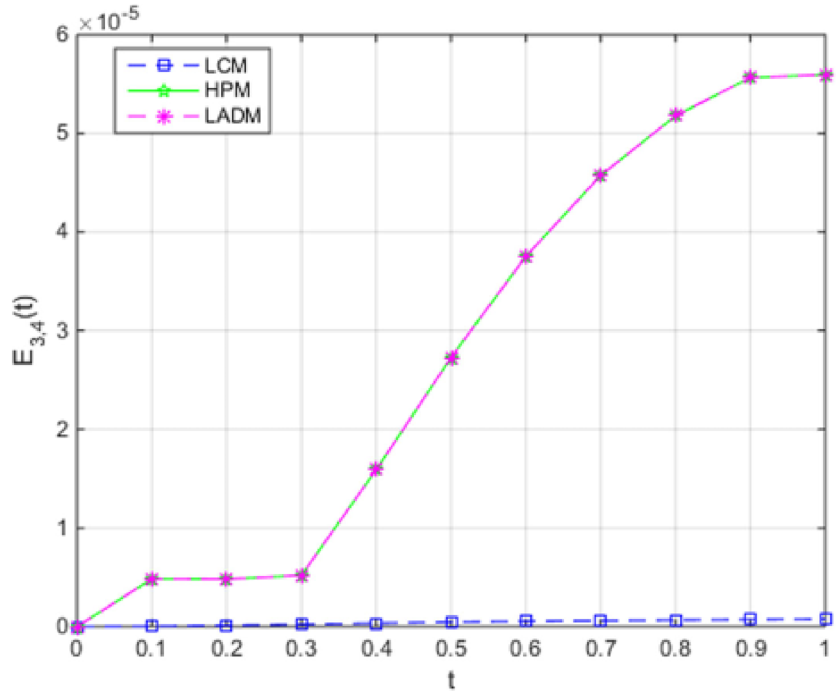

Fig. 4 Comparison of the error function for Laguerre Collocation Method (LCM), Homotopy Perturbation Method (HPM) and Laplace-Adomian decomposition method (LADM) for $u_3(t)$, $N = 4$

tion Method (LCM), than the other techniques, which are i.e. Homotopy Perturbation Method (HPM) and Laplace-Adomian decomposition method (LADM). We have the output in the time sub-interval when we investigate the dynamics. Similarly, HPM and LADM have stable error function results for t values between 0.1 and 0.3, while LCM has also stable results in the same time period. Moreover, it is seen that our method's error is less than one or the other techniques.

We can have an idea about the approximations with respect to the comparisons of the error functions of $u_1(t)$ and $u_2(t)$ in Figs. 5 and 6. These comparisons of both functions are shown for $N = 8$ and $N = 10$.

In Figs. 7 and 8, we note that the sensitivity analysis results for $u_1(t)$, $u_2(t)$, and $u_3(t)$ functions with the same initial conditions $u_1(0) = \lambda_1 = 5$, $u_1(0) = \lambda_2 = 5$, and $u_3(0) = \lambda_1 = 5$ and for the time parameter t which is given between $t = 0$ and $t = 100$. Even if we set the same number of ignorant, spreader, and stifler individuals at the beginning, the dynamics work and we have a different number individuals later on. This is exactly the information that we know from our deterministic model for the rumour propagation model with its given initial conditions. If we compare two figures, Figs. 7 and 8, we can see that β affects the dynamics and its sensitivity analysis slightly, which plays an important role as a constant in the model. In the sensitivity of ignorant individuals, we never have any negative results which shows us that the effect on time-dependent variables about these individuals does not change by time. On the other hand, we have the negative results for the spreaders and stiflers for their sensitivities. This argument of optimization shows us that they have an optimized results when the time parameter is increasing.

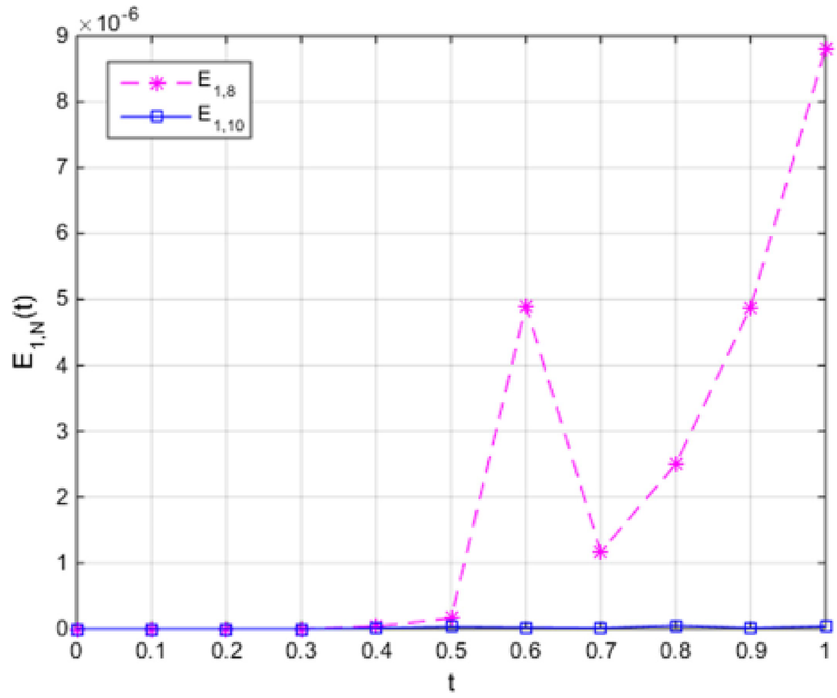

Fig. 5 Comparison of the error functions for $u_1(t)$, $N = 8$ and $N = 10$

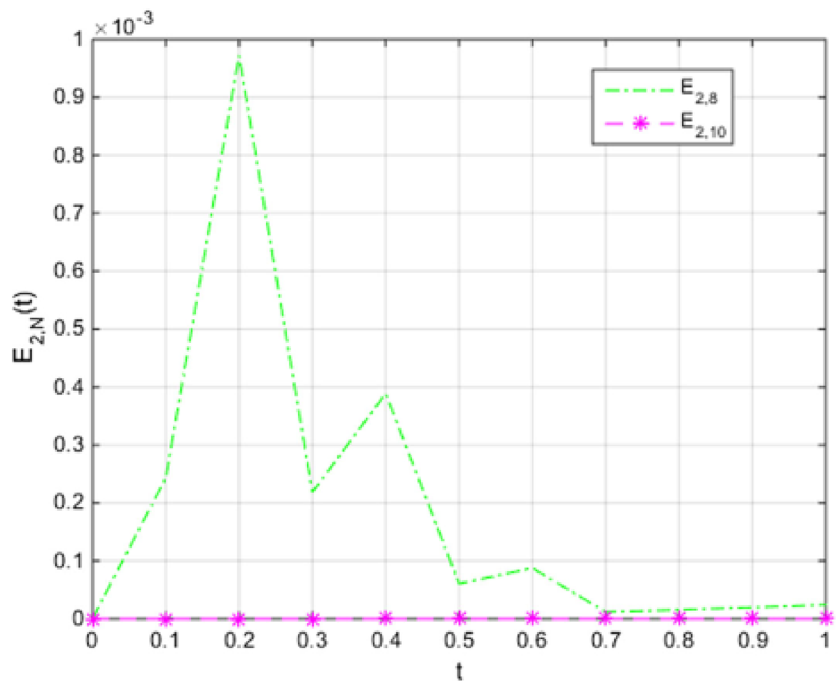

Fig. 6 Comparison of the error functions for $u_2(t)$, $N = 8$ and $N = 10$

The sensitivities of the ignorants, spreaders and stiflers with respect to the initial conditions at time t are given in Fig. 9. The qualitative behaviour of the dynamics is understandable from the sensitivity results in Fig. 9 of our model.

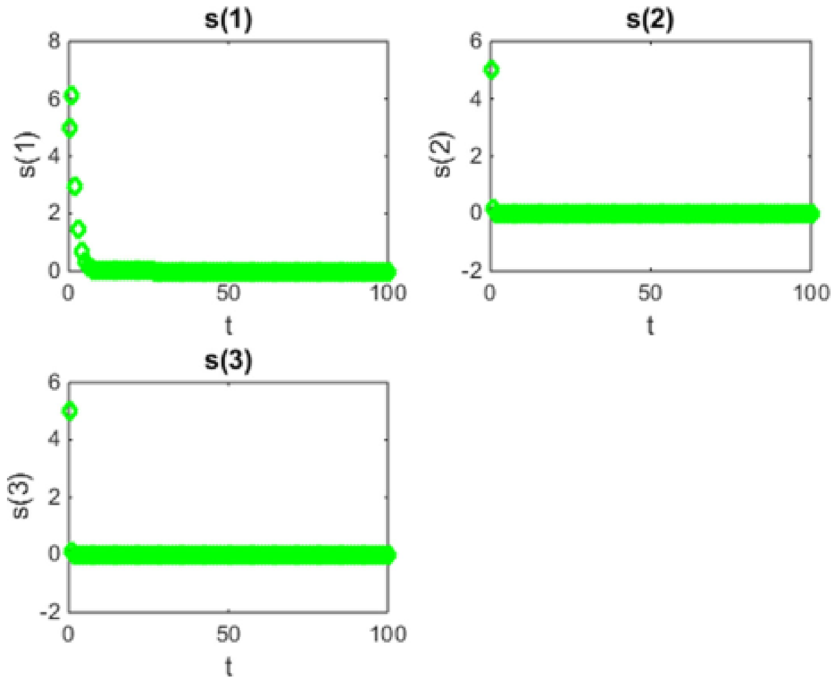

Fig. 7 Sensitivity results for $\beta = 0.01$, $\gamma = 0.02$, and $\lambda_{1,2,3} = 5$

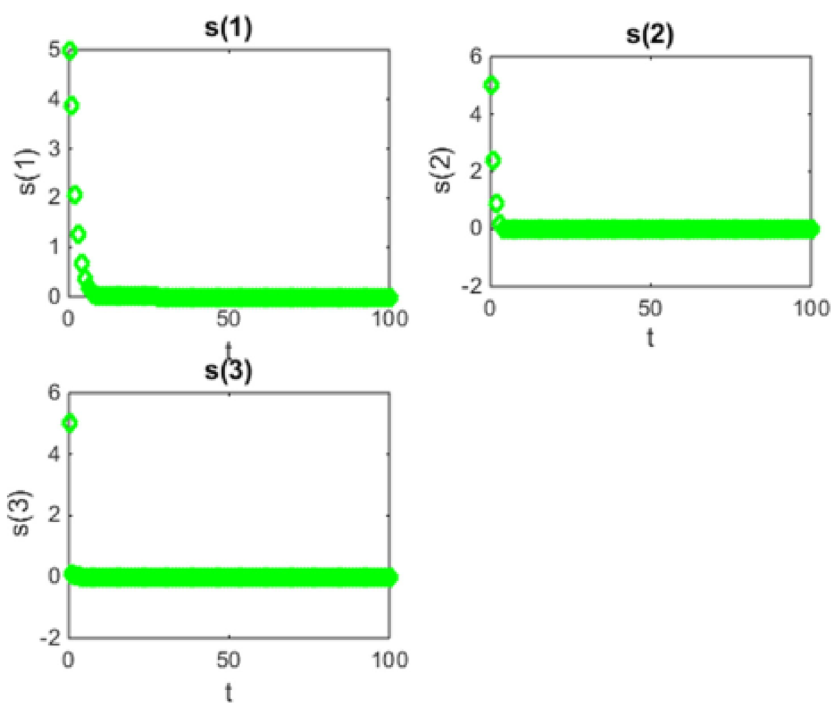

Fig. 8 Sensitivity results for $\beta = 0.1$, $\gamma = 0.02$, and $\lambda_{1,2,3} = 5$

6 Discussion

In this study, we deal with a novel rumour propagation model based on a modeling on spread with an approximation approach. Besides, the accuracy and sensitivity analyses have been investigated in order to show the beneficial results in our field of special

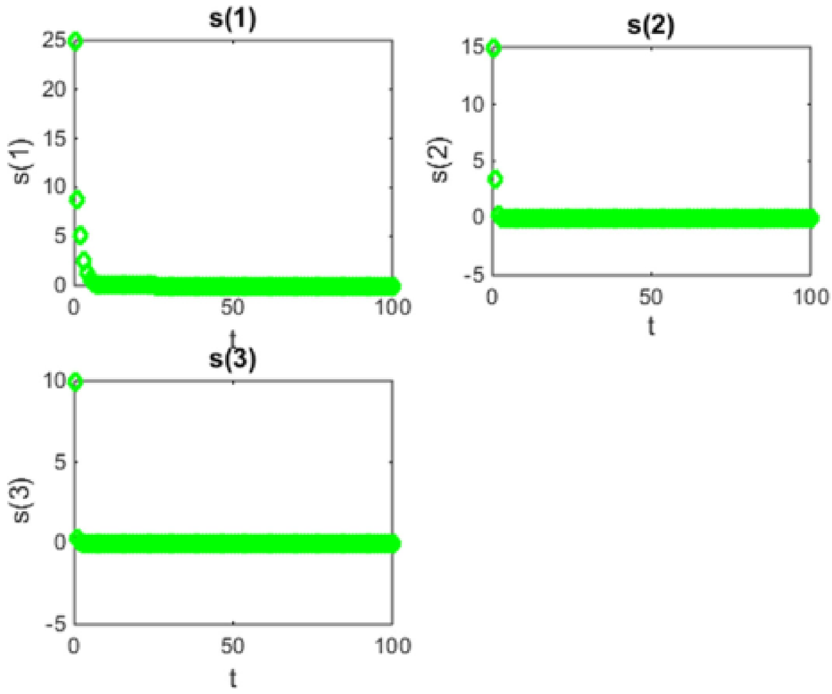

Fig. 9 Sensitivity results for $\beta = 0.1$, $\gamma = 0.02$, and $\lambda_1 = 25$, $\lambda_2 = 15$, and $\lambda_3 = 10$

interest. The algorithm has been logical and straightforward, and designed carefully and consistently.

We have numerical illustrations to reach rumour propagation model. We apply a technique in order to investigate approximations between some numerical solutions of the problem which are also compared with other techniques. Then we have reached for an error analysis in order to show the efficiency of the present technique. On the other hand, we consider the sensitivity analyses of the model with related to the parameters. These results are displayed the readers to give more understanding on the dynamical system comprehensively as well as open more interesting subjects to research later on.

We have some limitations to show more details in the dynamics due to our model. However, fundamental results have been achieved and implications of the study already show by our present and planned future modeling and investigation, we can successfully control accuracy and complexity of our new tool in applications on diverse OR fields. We wish to further advance and analyse in detail the model and limitations of our approach and, whenever needed, to overcome them.

7 Conclusion and outlook

In this study, we have obtained a rumour propagation model for optimization and real-life decision making based on the dynamics which we represented (Zhao et al. 2019; Zhang et al. 2012). The role of the parameters in the dynamics show us the interaction of spread of rumours by ignorants, spreaders and stiflers. This social phenomenon of rumour has been assessed by a novel numerical technique from Operational Research and Information Theory.

We have applied Laguerre collocation method on the ODEs system by considering a continuous time and given initial conditions. We have shown the efficiency and accuracy of the approximate solutions by the presented technique with an example. Tables and figures have demonstrated that the approximation results mean a valuable contribution as an alternative method in understanding information spread, for economic and societal practice. Moreover, the sensitivity analysis and the results have been visualised by figures. These results give us a better understanding of the model and the dynamics with respect to our accuracy approach and for the application of our novel technique.

We will advance, apply and post-process investigate any needed parameter estimation based on one real-world data in our future research work. Furthermore, we may turn to more explicit forms of time dependence in our differential equations, and regime-switching phenomena could be addressed as well. In forthcoming studies, our investigations be OR become employed and extended towards challenges of marketing, spread of new fashions, ideologies, but also of democracy and its novel trends and expressions.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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